Bethel University

Spark

All Electronic Theses and Dissertations

2024

How Sequencing Math Skills for High School Students With Math Goals in an Individual Education Plan (IEP) Supports Academic Growth

Harland Blake Field Eichmann
Bethel University

Follow this and additional works at: https://spark.bethel.edu/etd

Recommended Citation

Eichmann, H. B. (2024). *How Sequencing Math Skills for High School Students With Math Goals in an Individual Education Plan (IEP) Supports Academic Growth* [Master's thesis, Bethel University]. Spark Repository. https://spark.bethel.edu/etd/1097

This Master's thesis is brought to you for free and open access by Spark. It has been accepted for inclusion in All Electronic Theses and Dissertations by an authorized administrator of Spark. For more information, please contact k-jagusch@bethel.edu.

HOW SEQUENCING MATH SKILLS FOR HIGH SCHOOL STUDENTS WITH MATH GOALS IN AN INDIVIDUAL EDUCATION PLAN (IEP) SUPPORTS ACADEMIC GROWTH

A MASTER'S THESIS

SUBMITTED TO THE FACULTY

OF BETHEL UNIVERSITY

BY

HARLAND BLAKE FIELD EICHMANN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF ARTS IN SPECIAL EDUCATION

JUNE 2024

HOW SEQUENCING MATH SKILLS FOR HIGH SCHOOL STUDENTS WITH MATH GOALS IN AN INDIVIDUAL EDUCATION PLAN (IEP) SUPPORTS ACADEMIC GROWTH

BETHEL UNIVERSITY

BY

HARLAND BLAKE FIELD EICHMANN

APPROVED

PROGRAM DIRECTOR: KATIE BONAWITZ, ED.D.

THESIS ADVISOR: CHARLES S. STRAND, ED.S.

JUNE 2024

ACKNOWLEDGEMENTS

Thanks to my wife, Sheila, for your encouragement, many edits, and taking over the cooking to make this happen. Hallie for taking on more chores, Jac for making all the coffee, and Kaleb for making sure I was focused.

ABSTRACT

This thesis focuses on how developing standards-based high school math goals can facilitate meaningful collaborations with Individual Education Plan (IEP) team members. Math goal setting is a required part of many IEPs, but little research has been done to understand how to best set goals that are appropriate, in line with evidence-based teaching methods, and focused on the concepts most essential to create meaningful access to the general education curriculum. To investigate these questions, this thesis examines research into what skills and knowledge are required for students with disabilities to achieve high school level math. Being successful in math requires both procedural skills as well as a conceptual understanding of what numbers represent and how they relate to each other. Math goals should reflect and measure both of those components. Goals should also focus on moving students to the proximal domain of numeracy from additive to multiplicative to rational numbers in the same order as the general education curriculum progresses for students who are significantly behind in math.

TABLE OF CONTENTS

CHAPTER I: INTRODUCTION

AUTHOR'S INTEREST AND MOTIVATION FOR TOPIC

When students run into a difficult spot in math and must decide whether to persevere, there is no more common question asked than, "Why do I have to learn this if I will always have a calculator in my pocket?" It is a gleeful retort to the antiquated taunt of teachers that, "It's not like you will have a calculator in your pocket the rest of your life." While math education has clearly survived the transition from a computer being an occupation to a large clunky machine and even to a wristwatch, the question about what math a student really needs to learn becomes more urgent when the learning is difficult for a particular student while their peers move on. Then, the question is no longer rhetorical; it is asked by a family member to a teacher, "Why can't they just use a calculator to move past the basics?"

It is not just parents and frustrated students who wonder about appropriate math accommodations for students with learning disabilities. The calculator, as the physical embodiment of the debate over math accommodations, has been investigated for at least 40 years (Russell, 2014; Shult, 1978). More broadly, research spanning the last twenty years shows meaningful differences in the teaching and assessment styles between general and special education teachers when teaching students with disabilities (Maccini & Gagnon, 2006; Sheppard, 2020). The thesis author has taught middle and high school mathematics for three years and has faced these very questions and differing opinions from general and special education teachers, as well as evaluation team members. For example, is three-digit addition or two-digit multiplication an important part of mathematical learning or a perfunctory task that can be replaced? If a student does not know their basic multiplication facts, is it better for them to always have access to an accommodation? Although an accommodation would allow that student to progress, skipping skill mastery also makes it more difficult to use that skill in more advanced tasks, such as identifying greatest common factors needed in simplifying fractions or factoring algebraic expressions. Without multiplicative patterns in mind, factors and multiples are not obvious when a student is looking at an expression or equation; then finding the solution may require tedious trial-and-error repetition to uncover.

To make these important decisions, special education teachers must understand what high school students need to know to make meaningful progress in mathematical thinking and what deficits or gaps can be accommodated to enable a student to reach their potential. A sequential approach provides more “truths” which students can use to do more authentic thinking and learning about math to solve real problems in school or later in life. Understanding these properties of math enables the student to understand the operations the calculator is using to solve problems; for example, many students don’t have the idea that a calculator is breaking apart a number into exactly equal groups for division. Perhaps most importantly, a sequential approach provides a feeling of confidence from mastery. However, the sequential approach also comes with real drawbacks. This model is difficult to implement in a way that builds upon student strengths because they need to focus on weaknesses in order to progress. It is also hard to encourage high school students to engage in a curriculum that might be considered childish, and how students feel about the content they are studying is an important consideration since it can take much more energy to get a student to do a non-preferred task poorly than to do a preferred task well.

Contrasting the sequential approach is an integrated approach that continues to tackle mastery of basic numeracy skills while teaching more advanced content. Higher level content may not be discussed at some Individual Education Plan (IEP) meetings because it is considered outside of the student's zone of proximal development; however, is it possible that some of those basic number patterns and meanings would be able to be taught along the way in a more advanced curriculum? Integrating basic skills into a higher-level curriculum would provide access to higher level math skills, the ability to build on strengths instead of focusing on weaknesses, and a feeling of confidence born of doing math closer to grade level.

To look at these questions, this thesis will focus on the selection of appropriate math goals written into IEPs because a math goal is where mathematics educational theory becomes actualized into practice. It is where the mathematics plan is communicated to those questioning families and teachers. Math goals are the most frequently measured and reported aspect of an IEP; well-written goals also communicate a student's general level of mathematical ability. Other sources of information are available, but often that information is not as current or is too general to be actionable, such as "5th percentile nationally". Even though these facts can be useful for tracking improvement over many years, there is no clear sense of direction on how to progress from the 5th percentile to 12th percentile in the same way that there is to progress from being able to add like-fractions to being able to add unlike-fractions. An abundance of details can cause IEP team members to lose the forest for the trees and have no useful landmark for their student's progress other than their own vague memory of school. If families do have questions about math, they inevitably ask, "What grade level is that?" While this isn't the best way for teachers to conceptualize a student's unique set of skills and weaknesses, it is important to be able to effectively communicate a simplified picture of progress in an understandable way.

Math goals are also a part of the IEP that many special education teachers could improve by incorporating a better understanding of the progression of how math is learned. In the writer's experience of reviewing goals coming from many different schools and spanning grade levels from fifth to eleventh, many goals fail even the most basic standards of being specific enough to be measurable. For example, applied problems are a common math goal, such as to “increase her ability to solve two-step story problems including a mix of addition, subtraction, multiplication, and division from 40% to 80% accuracy.” There is too much variance in student ability and across math standards for that goal to be meaningful. For example, a two-step problem like, “Grace has $10 dollars. If she buys a pen for $2 and a candy bar for $1, how much money does she have left?” is very different from: “Calvin paints pictures and sells them at art shows. He charges $56.25 for a large painting. He charges $25.80 for a small painting. Last month he sold six large paintings and three small paintings. How much did he make in all?” The goal does not convey enough information to distinguish between the two questions.

But even goals that meet the minimum requirements of being a goal may not be an effective part of moving a student forward in their education. Goals that are specific enough to be measured meaningfully often miss the student’s overall progress. Many math goals focus too narrowly on one specific skill or select a skill that is too tangential to a student's overall progress to communicate a student's general ability. For example, a goal to, “improve his ability to answer questions about the perimeter and area of squares and rectangles” is a specific standards-based skill, but not one that provides any insight into what skills the student has or is missing. Does the student struggle with addition, multiplication, or do they just get the words perimeter and area mixed up? Goals of this kind also negatively contribute to the idea that math is presented as a constant stream of unconnected facts and procedures to be memorized instead of a logical progression of deeper reasoning and exploration of numbers.

Goals that lack a sense of progression are also difficult to match to the annual timeline of the IEP process. Focus on a specific standard, such as, “will improve their ability to (x) from

60% to 80%" do not set an expectation for how this level of accomplishment relates to a rate of growth or how that rate of growth relates to general education standards for progress for one year. IEP's cannot be expected to make completely accurate predictions about a student's future development, but it is unfortunate to see students arrive at high school without any conceptualization of how their math skills compare to state standards. Without a larger view of a student's progress, families and students miss out on the opportunity to prognosticate their student's total math capability by the time they graduate high school. When families ask if their student will be in ninth grade math next year for a student with a third-grade math goal, this isn't because families think their student will learn six years of math in one year's time. This happens because they have not been given useful mileposts to make the tracking of the student's progress meaningful.

The researcher hypothesizes that the above-listed problems identified in current math goals may be in large part due to special education teachers' lack of understanding of how mathematical sense-making develops, and therefore, how to best chart its progress. Unfortunately, it is well documented that special education teachers lack sufficient training in, or even knowledge of, best theories and practices in mathematics (Maccini & Gagnon, 2002), and therefore, may not be confident in accessing those practices when making important decisions in the mathematics trajectory of their pupils. The author has first-hand experience completing all the coursework for a master's degree in special education without specific instruction in teaching mathematics, all while teaching four math classes a day. This is reasonably expected to be the same situation for most special education teachers since there are no state requirements pertaining to math for licensure in special education (Minnesota Legislature, n.d.). The need is clear for special education teachers to have research-based information when writing math goals.

Researchers have argued that educational practices would be improved by increasing collaboration between the fields of mathematics and special education, given the different knowledge sets by each group of teaching professionals (Sheppard & Wieman, 2019) so the purpose of this thesis will be to take a step towards applying mathematics to special education.

GUIDING RESEARCH QUESTIONS

1) **Primary Question:** How does sequencing skills for high school students with math goals in an IEP support academic growth?
2) **Secondary Questions:** Is learning math necessarily sequential, or can there be multiple entry points to the same concepts? Are students more successful when given higher-level content with increased accommodation or when focused on numeracy until fundamental operations and computational sense are mastered? For students with disabilities, is it more effective for math to be taught using different approaches than in general education, or should math for students with disabilities be approached in fundamentally the same way at a developmentally appropriate level with effective accommodation? Bringing this information together will shed light on how to best develop IEP math goals, as well as how to map out a student's math curriculum trajectory.

CHAPTER II: LITERATURE REVIEW

RESEARCH PROCESS

Academic literature researched to answer these questions comes from articles available through the Education Resources Information Center, the ProQuest Education Journals Database, and Google Scholar. Those databases were searched using the terms: math*, accommodat*, learning disabilit*, special education, goal*, numeracy, embedd*, geometry, interventio*, secondary school*, IEP, sequence, curriculum, content, high school, integrated math, and best practices in a variety of combinations. Additional relevant research was uncovered through the references of articles.

LITERATURE REVIEW

Looking first at the sequencing of math goals, Xu et al. (2023) undertook the challenge of quantifying the relative importance of each of the major steps in mathematical reasoning in a person's ability to successfully achieve the next skill. To do this, the authors distilled math reasoning into five broad categories: fundamental numeracy, additive relationships, multiplicative relationships, rational numbers, and algebraic knowledge. It is worth noting that not all researchers use the same categorization of math knowledge, and this list does not include areas of spatial reasoning or data and probability; for this reason, the results are focused on what is undeniably the bulk, but not the complete body, of primary math education: number, operation, and algebra.

This was a quantitative study that included 236 college students who completed math tasks in all of the defined areas and the scores in each category were analyzed for significant relationships with each of the other categories (Xu et al., 2023). The participants were all volunteers from the principal researcher's university who completed the test and affirmed at its conclusion that they had tried their best to answer each question. The authors hypothesized that the most significant correlation found for each area was the participant's performance on the immediately preceding step in the order listed above. Scores for the four basic operation questions were based on the number of seconds required to correctly respond, while scores for the two most complex sets of tasks, fraction arithmetic and algebra, were scored based solely on correctness. The results supported the hypothesis. For example, the overall correlation to success in algebra tasks was 50% stronger between success with rational numbers than with multiplication. The relative importance of the immediately preceding step became even more pronounced when each category was examined for unique variance by controlling for the difference in the other skills. Then, the unique variance for success on algebra tasks that could be attributed to skill with rational numbers was 17.2%, compared to 1.2% for multiplication.

Xu et al. (2023) also attempted to control for differences in working memory by including a reverse sequence memory task. This does not mean that there are no differences in how students with disabilities learn (Powell et al., 2013), as will be examined with greater detail later, but it does suggest that the findings of Xu et al. (2023) are universalizable to students both with and without mild cognitive disabilities. Although, as the authors admit, this suggestion is limited by the inclusion of only one potential cognitive impairment in the study.

So, evidence supports the idea that mastery of each math skill, at least in the broad categories of number, four operations, and algebra, is correlated to procedural skill in the preceding step (Xu et al., 2023). The ability to order numbers is predictive of ability to add them together, as adding is to subtracting, and multiplication ability is correlated to division ability. The positive correlation between the preceding skill to the measured skills continues through rational numbers and algebra. There is a difference, however, in that first relationship that does not reappear in any of the later steps. Numeracy, measured as ordinality, is a conceptual understanding that was shown to translate into a skill proficiency, measured as response time to single-digit addition. Only one of the steps measured, numeracy, was a concept instead of a skill. The study successfully added evidence to the case that fundamental skills need to be reinforced for students who are struggling with higher level skills. Do those same fundamental skills also support more complex conceptual understanding?

Namkung et al. (2017) provided evidence that, in at least one important step, proficiency in fundamental operation skills was indeed correlated with conceptual understanding of the value of fractions. Namkung et al. (2017) used a quantitative study of 1108 fourth-grade students from 14 schools to compare the skill level in whole number, four operation questions, from single digit addition to multi-digit multiplication with regrouping, using the relevant subset of questions from the Wide Range Achievement Test-4 (WRAT-4). They compared performance at the beginning of the year with the students' conceptual understanding of fractions at the end of the fourth-grade. Conceptual understanding was measured by the ability to transfer symbolic representation into pictorial representations and vise-versa, to place fractions on a number line, and to compare fractions with different denominators. In short, students were not tested on their ability to perform mathematical operations using fractions, as in the study by Xu et al. (2023); rather in this study, students were tested on their ability to represent the cardinal and ordinal magnitude of fractions.

Namkung et al. (2017) predicted that students without adequate whole number competence would be at a significantly greater risk of not understanding fractions, even though fractions are a distinct representation of value from whole numbers. The research confirmed the hypothesis and found that 84% of students in the bottom 10th percentile of whole number skill end the fourth-grade year with difficulty in understanding fractions, compared to only 47.26% of students who scored between the 10th and 25th percentile and only 18% of students above the 25th percentile. This equates to students with significant whole number operation difficulty being 32 times more likely to also struggle significantly with comprehending fractions. While at first glance, this hypothesis may seem like a basic assumption that students for whom math is difficult continue to struggle with math over time, there is a useful implication for what kinds of interventions may help these students because of the potential connection between math skills and conception of magnitude. Although the authors do not make this connection, the finding is in line with a cognitive load theory explanation (Paas & Van Merriënboer, 2020) that facility in whole number operations reduces the complexity of comprehending new values in new symbolic representations.

As may be predictable now based on Xu et al's (2023) finding, conceptual understanding of magnitude and operations follows the same pattern. Just as the previous study (Namkung et al., 2017) found evidence for the conceptual link between whole number magnitude and fractional magnitude, Booth and Newton (2012) found the understanding of fractions to be key to being successful with algebra. In line with the previous study, understanding of the most adjacent concept, rational numbers, had a much larger correlation to understanding algebra concepts than whole number place value, lending support to the importance of fractions in understanding algebra. The researchers tested understanding of magnitude by having middle school students place whole numbers and fractions on number lines. For the fractions, the number line was limited to zero to one, because while there are at least five ways students must understand fractions, a number line from zero to one demonstrates a fraction as both a part of a whole and a single number with a discrete value. To measure algebra concepts, this study asked students to state the meaning of algebra symbols, to solve one step equations, and to represent word problems as algebra equations with a single variable.

Interestingly, Booth and Newton (2012) also found measurable differences in correlation to algebra concepts between unit fractions and non-unit fractions. Unit fractions are any fraction with a numerator of one, representing one unit of that denomination. Non-unit fractions are those with any numerator not equal to one. This was not a research question they intended to investigate, so there is no specific conclusion drawn in the paper. It is worth mentioning because the researchers hypothesize that the difference may be due to a difference in how students conceptualize unit and non-unit fractions. Placing non-unit fractions on a number line uses proportional estimation, identifying the proportion of the denominator represented by the numerator. This is a valuable skill, but non-unit fractions reduce the need to accurately conceptualize and estimate the magnitude of the fractional unit, which is best measured by the ability to accurately plot unit fractions. The authors only examined the correlation, and not the causation of the link between fractional numeracy and algebra concepts. The authors left it open to future research to determine the cause of the relationship. It could be based on a reduced toll on working memory for students learning algebra when fractional symbols and meaning are familiar and automatic. It is also possible to be an advantage gained by a thorough understanding of number magnitude that is demonstrated better in fractions because they are more complex than in whole numbers.

The research reported above looked at how all students learn math and reinforced the importance of fundamental math skills as building blocks for more complex math. Powell et al. (2013) reviewed the last thirty years of literature on how students with identified learning disabilities, as well as students who struggle with math without an identified disability, best learn math and compared this body of knowledge with common core standards for math. The authors looked at 15 studies of effective interventions for students with math difficulties from between 1997 and 2009. They found points of difference and similarity between the emphasized knowledge in common core math standards and evidence-based practices for students with math difficulties and attempted to outline entry points for teachers to provide access to grade level curriculum for students with math difficulties. The authors found that as a student progresses through school years, math standards are moving towards a deeper conceptual understanding of math as well as problem solving, with less emphasis on explicit instruction of step-by-step mathematical procedures. However, evidence-based practices support the use of explicit instruction for students with learning disabilities in math. Math difficulties have been shown to be linked with deficits in both working memory and semantic memory (long-term memory of words, concepts, and numbers) that make complex, higher-level math skills more difficult to comprehend without being broken into discrete steps. Areas of alignment between standards and evidence include interventions that utilize manipulatives and representations of value to support understanding of magnitude, place value, and basic operations.

Other researchers warn against utilizing some methods, even those that have been found to be evidence-based to improve math scores. If those methods fail to substantially center the student as an active nexus of mathematical sense-making. In other words, explicit procedure instruction that lacks reasoning or explanation is a disservice to all students and could represent a structural inequality if it is only applied to a subset of students who are already struggling in math. Tan et al. (2022) argued that these methods are dehumanizing to students because they remove the opportunity for students to fully participate. Full participation includes a student bringing their own prior knowledge, experiences, and reasoning abilities to construct connections to new learning in ways that derive correct conclusions or answers in a way that makes sense to the student. Focus on direct instruction, especially in the context of students learning below grade level skills reinforces a negative mathematical identity that the student need not or cannot understand mathematical thinking.

For example, as Ballin et al. (2022) pointed out, there is more than one way to correctly, systematically, and efficiently derive the answer to a two digit by two-digit addition problem (see figure 1). Adding by place value from right to left may be the standard algorithm, but moving left to right is how students are already trained to read words or numbers, so switching to right to left may not only add an unnecessary barrier, but reduce that student's ability to effectively use important skills of generalization and estimation. Estimation is the ability to see the big picture and is an important skill throughout all levels of math to ensure that a precise answer is reasonable. Estimation, therefore, requires focusing on the largest place value first, reading left to right. Far worse, if it never makes sense why multidigit addition must or even can be done right to left, then it could even create a lasting fissure in that student's mathematical identity and ability to construct new knowledge later based on what they already know about place value. This is not to

Left to right	tfel ot thgiR
59	1
+17	59
60	+17
+16	76
76	

Figure 1 showing substantially equivalent ways to add multi-digit numbers argue for a change in the standard algorithm of multidigit addition. The standard algorithm is useful and understandable to many people. Rather, the purpose is to highlight the important difference between student-centered mathematical sense-making and explicit instruction in procedures that are centered on the teacher's understanding of math. The example above of multidigit addition is just a single example of substantially equivalent procedures possible for any math operation.

Tan et al. (2022) looked at published research studies on math education for special education students between 2007 and 2016 and found a trend away from curriculum and instructional practices that focus on direct instruction and towards constructivist methods. Of 61 the studies examined, 14 related directly to curriculum and instruction. Of those 14, 8 were found to study what the authors refer to as "humanizing mathematics education" while only 4 relied on practices that the authors argued removed the student's problem-solving creativity and replaced it with inflexible procedural steps that are "dehumanizing" as defined above.

In fact, this shift in understanding about the role of standard algorithms in math education can also be seen in the 2022 Minnesota Standards for Mathematics. The standard algorithm is included for seven operations in the current Minnesota Standards for Mathematics for third through eighth-grade (Minnesota Department of Education, 2008), but it is only mentioned once in the new standards for the same grade range. Even that single mention of the standard algorithm is limited to an option: "knowledge of place value and the properties of operations that may include partial quotients and standard algorithms" (Minnesota Department of Education, 2023; p. 57, standard 5.3.5.2). These considerations about uplifting the student and humanizing their experience of learning, combined with some counterfactual evidence but an overall lack of robust research for advantages for teaching students with math difficulties in substantially different ways, it is reasonable to conclude that IEP math goals should not be written in a way that prioritizes specific procedural tasks at the expense of more conceptually robust mathematical content.

In fact, a review of 14 different systematic reviews, referred to as an umbrella review of math learning disabilities funded by the Australian Department of Education came to a similar conclusion (De Bruin et al., 2023). The systematic reviews examined were written between 1999 to 2022 and encompassed hundreds of individual, peer reviewed primary research papers. The review looked at what teaching practices had the most compelling research evidence for students with identified learning disabilities and found that while the amount of time needed or the setting may be different for students significantly behind in math, the most effective methods and interventions are the same effective practices used with all students: using concrete and visual models, graphic organizers, explicit and graduated instruction with guided and independent practice, and corrective feedback. The implication of this extensive review is that it confirms that IEP goals should not only be standards-based on content, but that they should also focus pedagogically on similar forms of instruction. This means math goals should go beyond memorization of facts and lists of steps to engage with problem solving, conceptual understanding, and flexibility.

The research has shown that many math skills and concepts build on previous knowledge in a way that is well documented in correlation between the broad steps of mathematical thinking. All students, including those with significant math difficulties, learn best with access to mathematics instruction that allows them to be “confident in themselves as doers, knowers, and sense makers of mathematics” (Huinker et al., 2020, p. 23).

Studies that look at how math goals are written, however, find the opposite to be true. Two studies that looked directly at existing math goals found that the majority of goals for middle and high school students focus on basic operation procedural skills. Hott et al. (2020) looked at IEPs from 15 school districts from the Southeast United States. 89 IEPs from secondary students with needs identified as mild were reviewed in the study, which found that 88% of the needs identified were basic calculation skills and less than 20% specifically referenced algebra, geometry or statistics. The authors noted a lack of goals related to concepts known to be essential to success in standards-aligned algebra, notably the same concepts highlighted in this thesis: rational thinking and problem solving. The researchers concluded that districts and their students would benefit from specific training for administrators and special education teachers for better alignment of student goals and services for math needs, suggesting that current practices are not sufficiently rigorous.

Scanlon (2013) did not directly review and categorize IEP goals like the previous study. Instead, questionnaires were sent to school districts in Delaware. Seven school districts and 3 independent schools responded with a representative mix of large and small districts from Delaware's total of 19 districts. While this study found many more districts, 60%, explicitly reported writing needs based on grade level standards, the vast majority, 70%, wrote goals based on computational skills to measure progress. Eight of the respondents said that they offered no resources to help special education teachers select appropriate, evidence-supported math goals. Also alarming, none of the responses from districts or schools showed that goals were connected in a clear progression of learning pointing towards access to grade level content. So how do educators bring together the need for conceptually-rich math instruction for students who lack the fundamental skills that have been shown to be necessary prerequisites for progress?

One potential way to accommodate for deficits in learning while also offering meaningful access to standards-based curriculum is to embed reteaching of foundational numeracy skills in high school level algebra lessons, as tested by Clausen (2022). Clausen used a case study model that included two high school students who received math instruction in a special education setting. By modifying the curriculum for these students, the researcher was able to provide access to challenging algebra concepts while also continuing to engage the students' need to master early numeracy skills, (whole number magnitude in this study.) Unfortunately, her research did not show a significant increase in the students' numeracy understanding or algebra problem solving, but this does not mean the approach does not show promising signs. First, the case study only included two students so it wouldn't be possible to consider any result conclusive. Also, other measures did show improvement. Teachers reported that both students showed increased participation and enjoyment in the intervention lessons. Importantly, survey responses from teachers and parents showed no detrimental effects of providing access to standards-based algebra concepts, even if the students were not able to demonstrate mastery by the end of the study. Providing the opportunity in this situation is consistent with the criterion of the "least dangerous assumption" developed by Donnellan (1984).

The least dangerous assumption is a guiding principle that, in situations of uncertainty about how to best educate students with disabilities, it is best to choose the alternative that will do the least harm to the student. Applied to math instruction, there is still much that we do not know about what students with math difficulties can learn and how to best present it (Hughes et al., 2023), even 40 years after Donnellan first introduced the idea. The alignment of embedding numeracy into algebra instruction with the least dangerous assumption about student's abilities, interest in the curriculum from parents and teachers, and the mandate from IDEA to provide access to the general education curriculum, combined with the limitations of having only two case study participants, led Clausen to believe that this practice could still be an area for fruitful research in the future.

Another way to move past basic computational goals is to include conceptual understanding in goal measurement (Scanlon, 2013). Scanlon reviewed existing research on what mathematical concepts underpin the successful use of fractions for the purpose of designing IEP goals and assessments that measure both skill and conceptual understanding. Including the conceptual understanding in the measurement opens up multiple entry points to the concept and enables teachers to leverage existing knowledge, building on strengths and humanizing the process for the student, as described by Tan et al. (2022). The five constructs of fractions identified by Scanlon in the literature are: part of a whole, measure, division operator, quotient, and ratio (order and equivalence are two of the first concepts that students learn about fractions, but they are both properties of fractions, not unique definitions.) For example, one student may conceptualize $\frac{1}{4}$ as a quarter of one, as a coin, or as a portion of many individuals in a group. All are valid entry points and most secondary students struggling with basic fraction operations will have at least some partial constructs of $\frac{1}{4}$ by the time they reach high school. Measurements that are specifically designed to parse what constructs a student is fluid in and what they have not yet mastered both validates their existing knowledge as well as elucidates what concepts they may need in order to access more complex concepts in the future.

Scanlon tested the effectiveness of using this form of assessment to parse unique levels of understanding between students by giving the assessment to 140 students spanning fifth-grade general education students, inclusion classrooms, and special education small group classes for seventh and eighth-grade students. Students with identified disabilities demonstrated significantly less understanding of fractions than students without disabilities and more importantly, the non-computational skills assessment showed unique variation between students that would not be demonstrable in computation assessments. This is significant because the crucial role of fractional understanding in mastering algebra has been shown above, so goals focused on numeracy may provide more meaningful access to the general education curriculum than skills-based goals.

Effective IEP goals rely on more than just a robust understanding of math. The Individual Education Act, (IDEA) mandates and research supports the participation of secondary students in the IEP process (Individuals with Disabilities Education Act, 2004). Fuchs et al. (1989) investigated the effect of student participation in setting learning goals specifically with high school students with learning disabilities. In the study, 20 students were randomly assigned to either choose a personal goal for level of mastery or be assigned a goal on a computational task with a mix of addition, subtraction, multiplication and division questions, up to two digits, measured at regular intervals over a three week period. Students who chose their own goal saw a significant increase in their performance on the computerized test, with a standardized effect size of 0.41, indicating a moderate effect. Interestingly, the researchers did not find a meaningful difference in the difficulty of goals set by students and by teachers. Both groups chose similarly challenging goals and the results looked at total correct answers, not whether a student reached the initial goal, so what goal was selected did not appear to be a factor in the results. The study also measured the effect of having a small reward (time playing a video game) based on whether or not the student met their accuracy goal. Being offered the reward did not show a significant difference in either the self selected or assigned goal groups, which shows just how powerful a sense of agency can be in student achievement. The researchers did note that the effect was most prominent in the first half of the three week period. There was limited evidence that the decrease over time was not due to a change in the assigned group, but a waning of the positive effect in the self selected group. Either way, the effect is unlikely to be as prominent in goals that are only reported at eight or twelve week intervals at school. The size and duration of this study was very limited, looking very specifically at the math goals for high school students with learning disabilities, but it does point towards student participation and choice being meaningful factors in student learning related to goal setting and the finding is consistent with several more recent research studies about student choice, engagement and behavior in general (Royer, 2017).

Meaningful participation from students and families in the IEP process may remain elusive, however, regardless of the mandate in IDEA and research suggesting its benefits. Murzyn and Hughes (2015) used a case study method with questionnaires, interviews and review of documentation to gain the perspectives of the various members of an IEP team to get beneath the question of compliance and look at how decisions are made. Murzyn and Hughes interviewed administrators, general and special education teachers, students and families about how decisions were made concerning math placement in three instances, one in each setting: a rural, suburban and urban high school. In all three cases, families felt that they were not included in the decision-making process despite being present at the meeting. Importantly parents reported that they did not have adequate information to fully participate and did not fully understand the options available. The parents in the study included a range of years of experience with special education as well as personal educational backgrounds. Even an active researcher in the field of special education math reported on his frustration in having a meaningful influence on decisions made in IEP meetings for his own child (Tan, 2017), so the disconnect is unlikely due to the interest or capabilities of the parents involved.

Murzyn and Hughes (2015) found that not only parents felt limited in their role though; administrators and general education teachers responded that decision-making was left largely to the special education teacher and both groups also demonstrated a lack of knowledge about how placement decisions were made even though standard sources of information, such as standardized tests like the Measure of Academic Progress (MAP), state achievement scores, special education evaluations, and classroom progress were documented. Two parents reported that data about their student's progress was not shared with them in an understandable way at the meeting.

The authors conclude that special education teachers could be more proactive in including families by providing them more resources to help them understand not only special education in general, but also their own student's situation in particular. They encouraged special education teachers to engage in "open and honest discussions about student needs and expectations" (Murzyn and Hughes, 2015; p. 55). Tan (2017) suggested that meaningful participation by families include, among other elements, an understandable table of standards-based conceptual understanding that creates a shared understanding by the whole IEP team of where the student is located in order to develop appropriate goals.

To that end, the application of this thesis is to create easy-to-understand presentations of the 2022 Minnesota Academic Standards in Mathematics that Minnesota schools will begin transitioning towards in the '25-'26 school year. These presentations will be applicable to high school students with IEPs, their families, as well as special education teachers.

CHAPTER III: APPLICATION OF RESEARCH

APPLICATION PLAN

In order to create a presentation of the 2022 Minnesota Math Standards that is both accurate to the original form and also accessible to an audience broader than just math teachers required arranging, summarizing, and finally depicting the information in a graphic presentation. **The first step** was to organize the specific benchmarks by mathematical anchor strand so that the progression of specific concepts is easier to visualize across grades. The standards are organized into three strands: data and probability, spatial reasoning, and patterns and relationships; those strands are then divided into seven "anchor strands" which are more specific: data sciences, chance and uncertainty, measurement, geometry, number relationships, equivalence and relational thinking, and patterns and relationships. All of the benchmarks, each of which is a "specific knowledge or skill that a student must master" (Minnesota Legislature, 2023, p. 1), are organized into one of the above anchor strands and then listed by grade. Organization by grade is ideal for teachers who teach a single grade, but would not as readily serve special education teachers who must work with students across a wide range of grade level abilities. For them, seeing each anchor strand connected across grade levels communicates clearly the growing complexity and nuance of a single strand through the years. So to do this, the Minnesota math benchmarks were organized into a second dimension, with grade level transposed to the columns and anchor strands as the rows.

This was an effective way to arrange the full-length version of the standards, but there was more clarity to be gained by further dividing each anchor strand into further subdivisions; this aided the same goal of narrating a compelling story of what a student is learning through time. This is especially true for the strand with the most skills and concepts, number and relationships. The number and relationships anchor strand contains benchmarks for place value, magnitude, number concepts, addition and subtraction, multiplication and division, fractions, contextual situations and finance, so further division into those categories enabled the readers to trace a single skill, like multiplication, from single digits through fractions to scientific numbers over the course of six years. The final part of the arrangement was to focus only on standards from third to eighth grade. The scope of the project was high school students with math difficulties, so beginning with third grade pertained to most students, except those with severe, low-incidence disabilities. It was sufficient to only focus on standards through eighth grade as any student who has already progressed through the eighth-grade standards has access to the grade level standards in mainstream classes.

The second step was to summarize each benchmark into a shorter statement. Summarizing each standard came at the expense of much of the detail and nuance from the original statements. However, the intent of the final product was to create an overview resource to locate a particular student's current level of competency, not to facilitate lesson planning or instruction so a summary better serves this purpose. In order to consistently summarize the 216 benchmarks in the chosen grade band, priority was given to capturing the core skill, the size and kind of number that it applied to, and the degree of change or what made it distinct from the previous grade. A standard with a twenty- to thirty-word sentence was distilled down to five to eight words with simplified language to clearly communicate the main focus of each benchmark. These first two steps of the project resulted in a summary of all the individual benchmarks between third and eighth grade captured on a two-page grid **(see Figure 2 below)**.

Strand	Area	Topic	3	3	4	4	4	4	5	5	5	5
1 Data Analysis	Data 1	Data Sets	Patterns and Questions 1 Data Set	Make Predition, Estimate Accuracy	Compare Two Data Sets				Patterns in Multiple Data Sets	Qualitative / Quantitative	Mean, Median, Range	
		Collection & Prediction	Describe Collection	Effect of Data Size	Collect Data, Est. Accuracy	Effect of Collection Method						
		Visualize	tables, bar graphs, number line		tables, double bar, timelines, line plots, spreadsheets				Center and Variability: double bar, graphs & plots		Compare/Contrast Models	
	Chance 2	Number & Model			Outcome as 0 - 1				Frequency Table	Predictions 0 - 1		
		Outcomes	impossible, certain, likely, unlikely		Categorize events							
2 Spatial Reasoning	Mea-sure 3	Tools	Nearest 1/4 Unit		1/16th in, 1mm	Protractor						
		Units	Compare Units within System	Add & Subtract Whole Number Change ≤ $1	2 decimals ≤ $20							
		2D	Perimeter Polygon, Whole Number		Acute, Right, Obtuse Angles	Perimeter & Area of 2D Figures	Composite Rectangle Area Grid or Dot Paper	Rectangle Area Formula, Whole Numbers	Area Formula Parallelograms, Triangles, Composite		Estimate Area Polygon/Non-Polygon Grid or Dot	
		3D							Volume Unit Cube	Volume & Surface Area Composed of Cubes	Volume Formula Rectangular Prism Formula	
	Geom-etry 4	2D	# of sides Triangles to Octagons		Points, Rays, Lines, Perpendicular, Parallel	Scalene, Isosceles, Equilateral Triangles	Quadrilaterals: Squares, Rectangles, Trapezoids, Rhombuses, Parallelograms & Kites					
		3D			Create Cubes, Rectangular Prisms		Cube Nets		Prisms & Pyramids	Nets for Prisms, Pryamids, Cylinders & Cones		
3. Patterns & Relationships	Number 5	Concept	More & Less, 100, 1,000, & 10,000	Unit Coordination, Power of 10, 5 digits	10x Value of Place to Right				More & Less, 0.1, 0.01, & 0.001	10x Place to Right &1/10th Place to Left		
		Ordinality	≤ 100k, <,>,=	Unit Fractions	≤ 1M, Whole Number, <,>,=	Fraction 0 - 3, Model & Nmbr Line	2 Digit Decimal, Nmbr Line, Mode		Decimals, 3 Digits			
		Add & Subtract	Estimate & Compute 4 digits	NonUnit as Sum of Unit /2, /3, /4, /8	Estimate Whole Number, ≤ 1M	Fractions ≤2, with Models, /2, /4, /8			Estimate Fractions to Nearest Half	Unlike Fractions	Estimate Decimals	
		Multiply & Divide	1 - 12, Fluent 2s, 5s, 10s and square products	1 digit x Multiples of 10, 100	Estimate, Multi-Digit Whole Number	Fluent, 0 - 12	Multiply by 10, 100 & 1,000	Decompose to Multiply, 4 x 1 Digit	Multiply Two Multi-Digit Factors Efficient Strategy	Multiply Whole Number by Fraction	Divide Multi-Digit by 2 Digit Divisor with Various Remainders	Most Useful Quotient & Remainer
		Fraction	Model & Number Line /2, /3, /4, /8	1/2 & 1 as /2, /4, /8	Number Line & Equivalent to Whole Number, 0 - 3, /≤ 12	Multiply by a Unit Fraction	Multiply by n/n for Equivalent Fraction	Equivalent to Decimals at 4ths	Multiply or Divide by n/n for Equivalent Fraction	Equivalent Decimals, Thousands	Multiplication of a Fraction of Whole Number, a/b of a group of n	
		Contextual Situations	Multiply & Divide, Visual Models, 1 digit		Divide, 4 x 1 Digit	Multi-Step, 4 Op, Multi-Digit, WN, Relationship of Ops			Multi-Step, 4 Op, Multi-Digit WN, Inverse Ops	Add & Subtract, Positive Rational Numbers	Whole Number Divided by Fraction, Visual Model	Multi-Step Add & Subtract, Positive Rational Numbers
		Finance	Spending & Saving Goals Add & Subtract Whole Numbers		Financial Desions Income, Spending, Saving, Giving, 4 Op				Differences in Payment Methods, 4 Operations	Budget & Debt, 4 Operations		
	Algebra 6	Concept	Commutative and Associative		Distributive Property, Multi-Digit							
		Equations	Open Number Add & Subtract, 3 digit	Open Number Multiply & Divide , 0&1	Open Number Multiply & Divide Multi-Digit				Missing Value Add & Subtract Fractions			
		Expressions							ID Equivalent Expressions, ( ) & 4 Operations			
		Contextual Situations										
	Pattern 7	Function	1 Rule, 3 Operations		Additive & Multiplicative				Ordered Pairs, Graph, + Integers	Compare 2 Patterns, Table or Graph	Ratio Tables Whole Numbers	
		Visual Pattern	Extend Pattern		Rule for Number in Pattern	Generate Number or Shape Pattern to Rule			Coordinate Plane Quadrant 1	Develop Rule to Apply to Fignure n		
		Tables and Graphs										

Figure 2. Two Page Summary of Third to Eighth Grade Math Standards

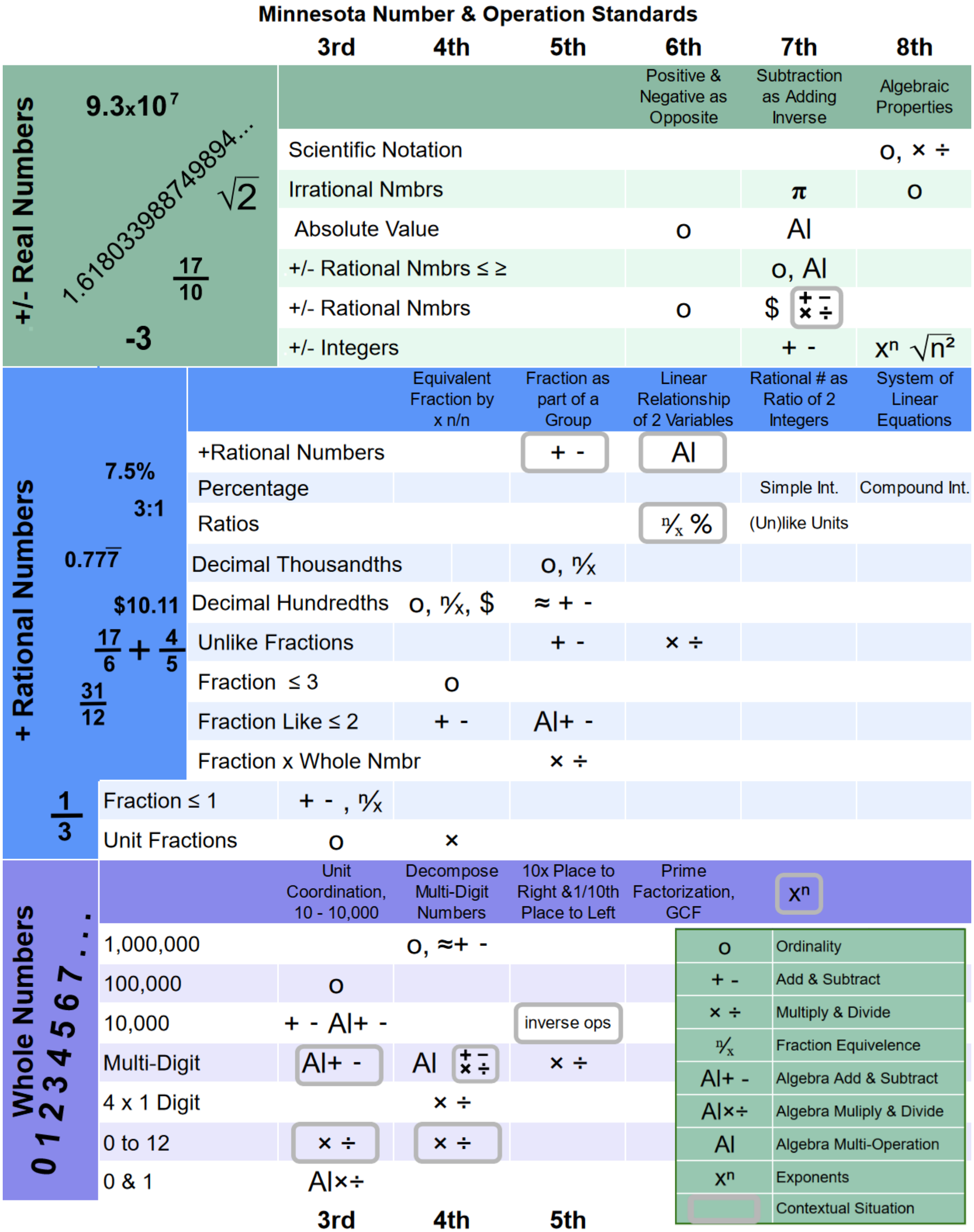
			6	6	6	7	7	7	8	8	8	8
1 Data Analysis	Data 1	**Data Sets**	Variability in Data	Explanations for Data Trends	Interperate Measures of Center & Variability	Sample vs Population	Compare Reasonable Alternate Explanations	Compare Center & Variability of 2 Samples to Populations	Association b/t Variables			
		Collection & Prediction	Design Data Collection			Samples to Compare Populations	Population Parameters based on Sample		Correlation vs Causation			
		Visualize	Create Visualizations Appropriate to the Data			Tables, Circle graphs, Histograms			Scatter Plot Bivariate Associations	Scatter Plot Straight Line Fit	Create Tables & Scatter Plots	Slope & Int Linear Model
	Chance 2	**Number & Model**	Sample Space Tree Diagram, Table	Ratio, Fration, %, Decimal		Probability as Fraction of Sample Space	Compound Probability as Fraction	Compound Probability List, Table, Tree Diagram				
		Outcomes	Experimental / Theoretical			Estimate Probability	Decompose & Simulate Compound Probability					
2 Spatial Reasoning	Mea-sure 3	**Tools**										
		Units	Convert within System	Estimate Units w/ Benchmarks								
		2D	Area Compose / Decompos			π	Circumference, Area Circles	Arc Area of Circles	Pythagorian Theorem	Pythagorian Theorem Missing Length	Distance 2 Points	
		3D	Surface Area Rectangular & Triangular Prisms	Volume Prisms		Cylinder Surface Area & Volume						
	Geom -etry 4	**2D**	Missing Angle Triangle	Interior Angles Polygons	Polygon Vertices on Coordiante Plane	Coordinate Plane Transformations/ Congruency	Similarity & Scale Factors	Scale Factors, Length & Area Ratios as Scale & Units	Slope-Intercept	Find Parallel & Perpendicular Lines	System of Linear Equations Solutions	
		3D										
3. Patterns & Relationships	Number 5	**Concept**	Positive & Negative as Opposite	+/- Rational Numbers on Line & Ordered Pairs	Absolute Value	Rational Number as Ratio of 2 Integers	Opposite signs are opposite sides of 0 & -(-3)=3		Rational & Irrational Numbers	+/- Integer Exponents		
		Ordinality	+/- Rational Numbers, <, >, =			+/- Rational Numbers, <, > , = , ≤ , ≥			Estimate Irrational Nmbrs on Nmbr Line	Scientific Notation		
		Add & Subtract				Subtraction as Adding Inverse						
		Multiply & Divide	Prime Factorization of Whole Numbers	GCF 2 Whole Numbers 2 digits	Fractions w/ Visual Models				Numbers in Scientific Notation			
		Fraction	Percent, Equivalent Ratios, Tape Diagram, Double Number Line			Ratios of Lengths, Areas in Like or Unlike Units						
		Contextual Situations	Estimate with Rational Numbers	Positive Rational Numbers, 4 Operations	Unit Rate	4 Operations, +/- Rational Numbers & WN Exponements						
		Finance				Budget +/- Rational Numbers			Simple & Annual Compound Interest	Multi-Step, Interest Rate, Time	Rate and Kind of Pay	
	Algebra 6	**Concept**	Ratios Different for Subtraction						Square & Square Root			
		Equations	1 Step Equations, +Rational Numbers			2 Step Equations/ Inequalities Rational Numbers, Graph			Multi-Step, Multi-Variable, Equations			
		Expressions	Create Equivalent Expressions, +Rational Numbers	Equivalent Fractions, Decimals & %		Equivalent Expressions, Rational Numbers, Whole Exponents	Expressions, Rational Numbers, Abs Value, Whole Exponents		Equivalent Expressions, Ass., Comm., Dist., Id., Inverse	Order of Operation, Radicals, Abs Value	Point-Slope, Standard Form	Compare 4 Linear Equation Forms
		Contextual Situations	Write Expressions, Equations, & Inequalities +Rational Numbers		Ratio & Rate Applied	Multi-Step Equivalent Ratios, Tables, Diagrams		Multi-Step Percent Financial Situations: Whole, Part, %	Multi-Step Linear Inequalities, Graph	Solve & Graph Abs Value Inequalities on Number Line		System of Linear Equations
	Pattern 7	**Function**	Linear Relationship w/ Variables			Ratio Tables, Equations, & Graphs	Proportional Relationships		Function as 1 Ouput, f (x) Ordered Pairs	Arithmetic Seq. as Linear Function	Geometric Sequence as Exponential Function	
		Visual Pattern				Proportional Relationship Table or Graph			Proportional/ Non-Proportional Linear Relationships		Linear/Non-Linear Patterns	
		Tables and Graphs							Linear Function Tables, Graphs	Slope-Intercept Graph	Slope-Intercept Properties	Arithmetic & Geometric Seq.

Figure 2. Two Page Summary of Third to Eighth Grade Math Standards

The final step was to create a single visual that accurately depicts how skills and number complexity develop over time and the relative pacing and expectations for growth over time. IEP goals do not need to match the pacing of general education standards, but understanding the general education standards is the right place to start when individualizing goals to meet the needs of a specific student. The graph focuses on anchor strands 5 and 6: "Number Relationships" and "Equivalence and Relational Thinking" because those strands contain the benchmark skills and concepts that were identified in the literature review to be most important in sequencing to a student's continued progress and mastery. Math goals for students more than two years behind in math should focus on anchor strands 5 and 6 for the same reason. For example, a student missing a fifth-grade skill in data sciences of calculating mean, median, and range, will not lose the ability to continue to progress and master other benchmarks outside of that strand. On the other hand, a student who is not yet competent in large number addition and division will not only need those skills to continue in number relations, but also to compute and make sense of mean, median, and range in data sciences.

The chart **(Figure 3)** was organized with the progression through numbers as the rows, with the smallest whole numbers at the bottom to the largest, most abstract numbers at the top. The rows were further divided into three broad categories: whole numbers, positive rational numbers, and positive and negative real numbers. Including these categorizations provided an even broader picture than the specific rows, enabling the information to be as accessible as possible to students and parents. To that same end, examples of each type of number were included on the left side of the page. Math skills were plotted on the chart by the numbers used for that skill at each grade level. For example, standards for addition and subtract were applied to whole numbers up to 1,000 in third grade, to like fractions less than 2 in fourth grade, and to

Figure 3. Chart of Third to Eighth Number and Operation Standards



positive and negative integers in seventh grade. As the literature review showed, it is important for a student's conceptual understanding to grow in step with procedural skill. The conceptual understanding of the numbers themselves is primarily measured through comparing and ordering numbers; these skills were labeled as ordinality on the chart, as well as estimation. Benchmarks of concepts not directly related to a specific type of number, including place value, equivalence, and properties of equations were included as well. These concepts were organized by the grade and broad category of number in the heading row for each category.

Not every family or IEP meeting will actively engage with the process of setting an appropriately challenging and sufficiently descriptive measurable math goal, but this chart provides an access point for families to be invited in and to engage at several different levels depending on their interest. At the very broadest, students and families can see a grade level equivalent to the present level of academic achievement and their annual goal. Looking a little more closely, families can see the types of numbers and skills their student will be learning and the full scope of skills before ninth-grade math. At the most detailed level, families can look at the particular numbers and compare the schools report to their knowledge of their child's use of numbers at home with measuring cups, fuel gauges, road maps, or shopping trips to fully participate in the process. And in rare situations where there is disagreement about placement or goals, this provides a standards-based foundation for finding agreement.

Whatever level of engagement is achieved in an IEP meeting, the chart will also be a helpful tool to special education teachers in writing math goals. First, in the upcoming transition to the new standards, this overview helps highlight concepts that have moved, such as the introduction of negative numbers and absolute value in sixth grade instead of seventh grade, and the application of positive and negative numbers specifically to creating a budget in seventh grade. The grade level of one skill will not be particularly important to a student with a math goal significantly different from grade level standards, but even in that case, the change in relative position between related standards will still help inform decision-making. For one example, adding and subtracting unlike fractions is now taught a year earlier than multiplying and dividing unlike fractions.

Beyond the transition to these new standards, this chart references sixty-five benchmarks across six grades to potentially use for setting appropriately challenging and sufficiently descriptive, measurable math goals. Most of the benchmarks charted fit into the six subcategories of: ordinality, addition and subtraction, multiplication and division, fractions, algebra, or contextual situations, all of which span at least five of the six covered grade-level standards. This helps in the selection of an annual goal that is appropriate because it is easy to compare the students trajectory to the grade level trajectory. It can also aid in writing goals that are descriptive enough to be measured by different people in different settings because the chart includes skills along with the numbers to apply them to for every referenced benchmark. Reviewing the previous example of a word problem goal that was too vague to be measurable (see page 9), the goal was written as, "increase her ability to solve two-step story problems including a mix of addition, subtraction, multiplication, and division from 40% to 80% accuracy." Using either the two-page summary or the one-page chart, the goal could be improved by assigning a level of number complexity along with the operations required, such as a third-grade benchmark of one-digit numbers with four operations, a fourth-grade benchmark of multi-digit whole numbers, or a sixth-grade level that includes decimals and fractions. For example, **a revised goal could be**: "Student will improve her ability to solve multi-step, contextual situations, from solving questions with addition, subtraction, multiplication, and division of whole numbers up to three digits with 80% accuracy to solving questions with addition and subtraction of positive fractions and decimals including mixed numbers and decimals to the hundredths place with 80% accuracy." This revised goal would contain enough information to be measurable and be traceable to specific state benchmarks. It also incorporates a comparable amount of growth for one year (fourth to fifth grade), if that is appropriate for the student. To help make the determination of whether it would be appropriate for the student, the graph shows adjacent skills, like already beginning to add and subtract like fractions less than two and identify decimal values equivalent to fractions up the hundredths. If those skills aren't in place, having an abbreviated reference chart helps keep the relationship between benchmarks in different areas in mind, so that goals are aligned with standards.

CHAPTER IV: DISCUSSION AND CONCLUSION

SUMMARY OF RESEARCH

To meet higher level math standards in the common core, students rely on multiple, specific math skills from up to 14 different strands of mathematical thinking. This means that difficulty with early math skills, even counting, through fifth grade was a strong predictor of difficulty with math through high school and adulthood. Unfortunately, the inverse is not an accurate indicator of math success because the introduction of novel concepts through high school may prove challenging for children who were previously successful. Students with math difficulties often lack the necessary fluency with those foundational skills, such as place value and operations, to both compute the operations and derive applicable meaning from the abstract mathematical representation. Therefore, to help students overcome these challenges, teachers should first identify the particular foundational skills required to achieve secondary math standards and then provide instruction that reinforces those fundamental elements. This process should incorporate evidence-based instructional techniques that support a deepening numeracy, such as concrete-representational-abstract (CRA) instruction.

Whether the math interventions with the most evidence support using different strategies with students with disabilities is still a contentious issue in the special education research community (Tan et al., 2017). While some have argued that explicit instruction focused on basic skills has been effective, others have argued that this approach is unnecessarily exclusionary of other ways of thinking and believe that the special education instruction should better match the shifts in understanding about how mathematics is taught in the general education classroom. This would include focusing more on conceptual understanding and problem-solving, as spearheaded by the National Council of Teachers of Mathematics (NCTM).

Xu et al. (2023), Namkung et al. (2017), and Powell et al. (2013) all confirmed the importance of both sequential instruction of math skills as well as proficiency in fundamental skills for students to successfully access higher-level math. The authors differed in how they conceptualized and categorized the numerous strands of mathematical thinking that must be combined for complex math, like rational numbers and algebra. This is in part because the first two studies aimed to establish a specific link between discrete skills and next steps while the last considered a more generalizable framework for conceptualizing math skills. However, the difference even between the two specific studies, as well as papers with the NCTM categorizations (Graham et al., 2018) does point to a lack of consensus among researchers about the most useful way to conceptualize the disparate elements of math education. This may be a fundamental step required to complete the higher order task of effectively sequencing math goals in individualized education plans.

Concerning IEP math goals in particular, researchers investigating current practices found significant room for improvement in the way that math goals are chosen and written. Researchers found that numerous school districts do not provide guidance on selecting effective math goals and there is often a lack of meaningful participation of other members of IEP teams, including families and math teachers. These practices are not in line with the mandate of IDEA to focus on access to the general education curriculum and include input from a diversity of sources, including families and students. Without student involvement in goal selection, special education teachers are also potentially missing the opportunity to increase student motivation through participation in their learning.

LIMITATIONS OF RESEARCH

Research into effective methods of instruction in special education is plagued with a large number of small populations that make broadly-generalizable findings difficult to obtain, and this thesis is no exception. Many of the included studies only looked at a single disability, while the total population of students that demonstrate math difficulties spans many different high-incidence disabilities, from Autism Spectrum Disorder (ASD) and Attention Deficit/Hyperactivity Disorder (ADHD) to Specific Learning Disabilities (SLD). This limits confidence in the conclusions of the underlying papers as well as their accumulated consensus if the underlying populations are different in a significant way. This is possibly one source of disagreement among researchers about how to best teach math to students with disabilities, as there may be underlying differences that are not being measured and reported. Similarly, studies that were designed to investigate the general population of students typically included only a single variable to control for differences in cognitive ability, such as working memory. Math ability is likely correlated to different cognitive processes in unique ways for which these studies did not effectively control.

IMPLICATIONS OF FUTURE RESEARCH

The development of thorough, mathematically-rich goals and objectives for IEPs is a comprehensive task as complex as curriculum development and it should be undertaken by leaders in the field instead of as an ad hoc process written by each teacher. It is possible that this has not been done up to this point because of the perception that such an endeavor would negate the individualization required to meet the needs of exceptional children. This is an artificial barrier though, because “individualized” does not mean completely unique. Individualized means that goals are appropriate to meet an individual student’s specific needs. It would not only be possible, but far more likely for special education teachers to accurately individualize goals based on student needs if a comprehensive set of priorities and goals was created by experts, guided by research.

Scanlon (2013) began this process for one major numeracy threshold, fractions. She identified sixty-three potential learning goals related to the ordering and equivalence of fractions. Future research should both narrow the specific skills and concepts about fractions most important to enabling student progress and expand that method to additional cornerstone concepts, such as place value and multiplicative reasoning. Winnowing potential goals from a list of all skills related to equivalent fractions or place value that a student has not yet mastered to a shorter list of the most important anchors will require a deeper understanding of how these math concepts relate to each other. Researchers have hypothesized plausible explanations for why whole number fluency improves fractional reasoning or whether fraction magnitude or unit coordination is more casually related to success in algebra and these possibilities should be tested to better inform the goal-making process.

PROFESSIONAL APPLICATION

In the fall of the 2024 school year, teachers across Minnesota will begin to familiarize themselves with the new state math standards (Minnesota Department of Education, 2022), just approved by the Minnesota Department of Education Commissioner in April 2024. Grade level teachers will have thirty to forty standards to review for potential changes and high school teachers will have less than thirty standards to review for each class. Meanwhile, special education teachers will work across a wide range of different grade levels, covering over two hundred standards to meet the needs of diverse students to create accurate, compliant IEPs. Special education teachers are in a unique position to benefit from thoroughly understanding how mathematical skills and knowledge develop over the course of many years. This project will help special education teachers conceptualize the progression of math learning, identify important milestones, and communicate clearly with families about their students' status and trajectory. It will also be a valuable reference for teachers over the next several years as they adjust curriculum to be prepared for full implementation of the new state standards during the 2027-28 school year. Having an overview of the standards will be a useful resource for creating a scope and sequence. It has already been utilized in planning meetings to ensure that different levels of math taught in special education math classes are providing a continuous progression towards grade level standards.

CONCLUSION

The mastery of preceding skills and concepts is strongly correlated with success in later skills; even areas of math that don't superficially appear directly related, like fractions and algebra, benefit from proper sequencing. This supports the practice of teaching math sequentially as developed in state standards, including to students who are more than two years behind in math. That creates a tension, however, between providing students with the opportunity to reach the prerequisite competency before progressing and the mandate to provide access to the general education curriculum. At least one study has looked at embedding basic numeracy skills into algebra lessons for high school students to bridge the gap between the needs of the student where they currently are and grade-level content. The intervention did not produce a measurable difference, so while more interventions designed around this same idea would need to be created and tested with many more students to make a fully-informed claim, there isn't an evidentiary basis that students will make more rapid progress if given higher-level content with increased accommodation.

Although several other methods of instruction have been shown to be effective with students with math difficulties, most of these methods have also been demonstrated to be successful with all students, leading to some dispute about whether there is or should be any difference in effective instruction between students with and without disabilities. The lack of large-scale, rigorous evidence for substantially different instructional methods, combined with the moral hazard of separate and unequal education is leading most researchers towards advocating for students with disabilities to receive the same, conceptually-rich, numeracy-based, problem-solving math instruction as all students.

To support the academic growth of high school students with math goals two or more years behind their grade level, math goals should embrace a deeper sense of mathematical thinking than focusing solely on operational proficiency by incorporating measures of conceptual understanding along with measures of mathematical operations. The skills and concepts selected for goals should match students' present levels and relate to the major conceptual steps shown to be significantly related to future progress so that goals are always aimed at grade level standards and ensure that the student is making connections between the abstract operations and their internal conceptualization of magnitude that fosters growth and makes mathematical thinking meaningful.

REFERENCES

Ballin, A., Davidson, E., Caron, J., & Drago, M. (2022). Making math add up for students receiving special education. *International Journal of Whole Schooling*, *18*(1), 1–28.

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, *37*(4), 247–253. https://doi.org/10.1016/j.cedpsych.2012.07.001

De Bruin, K., Kestel, E., Francis, M., Forgasz, H., & Fries, R. (2023). *Supporting students significantly behind in literacy and numeracy: a review of evidence-based approaches*. Australian Education Research Organization. https://edresearch.edu.au

Fuchs, D., Bahr, C. M., & Rieth, H. J. (1989). Effects of goal structures and performance contingencies on the math performance of adolescents with learning disabilities. *Journal of Learning Disabilities*, *22*(9), 554–560. https://doi.org/10.1177/002221948902200907

Graham, K., Burrill, G., & Curtis, J. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. National Council of Teachers of Mathematics.

Huinker, D., Yeh, C., & Marshall, A. M. (2020). *Catalyzing change in early childhood and elementary mathematics: Initiating critical conversations*. National Council of Teachers of Mathematics.

Individuals with Disabilities Education Act (Pub. L. No. 108-446, 118 Stat. 2647). (2004).

Maccini, P., & Gagnon, J. C. (2002). Perceptions and application of NCTM standards by special and general education teachers. *Exceptional Children*, *68*(3), 325–344. https://doi.org/10.1177/001440290206800303

Maccini, P., & Gagnon, J. C. (2006). Mathematics instructional practices and assessment accommodations by secondary special and general educators. *Exceptional Children*, *72*(2), 217–234. https://doi.org/10.1177/001440290607200206

Minnesota Department of Education. (2008). *2007 Minnesota K–12 Academic Standards in Mathematics*. https://education.mn.gov/mdeprod/idcplg?IdcService=GET_FILE&dDocName=005247&RevisionSelectionMethod=latestReleased&Rendition=primary

Minnesota Department of Education. (2022). Recommended Minnesota 2022 mathematics standards implementation timeline. In *https://education.mn.gov/MDE*. Retrieved May 4, 2024, from https://education.mn.gov/mdeprod/idcplg?IdcService=GET_FILE&dDocName=PROD081270%20&RevisionSelectionMethod=latestReleased&Rendition=primary

Minnesota Department of Education. (2023). *2022 Minnesota K–12 Academic Standards in Mathematics*. https://education.mn.gov/mdeprod/idcplg?IdcService=GET_FILE&dDocName=PROD070524&RevisionSelectionMethod=latestReleased&Rendition=primary

Minnesota Legislature. (n.d.). *Minnesota Administrative Rules 8710 - MN Rules Chapter*. Office of the Revisor of Statutes. Retrieved March 23, 2024, from https://www.revisor.mn.gov/rules/8710/

Minnesota Legislature. (2023). Minnesota Statutes, Section 120B.018, Definitions, Subdivision 3. Office of the Revisor of Statutes. Retrieved May 29, 2024, from https://www.revisor.mn.gov/statutes/cite/120B.018

Murzyn, A., & Hughes, T. (2015). Making mathematics placement decisions for high school students with high incidence disabilities: a collective case study. *American Secondary Education*, *43*(2), 44–58. https://eric.ed.gov/?id=EJ1064152

Namkung, J. M., Fuchs, L. S., & Koziol, N. A. (2017). Does initial learning about the meaning of fractions present similar challenges for students with and without adequate whole-number skill? *Learning and Individual Differences*, *61*, 151–157. https://doi.org/10.1016/j.lindif.2017.11.018

Paas, F., & Van Merriënboer, J. J. G. (2020). Cognitive-Load Theory: Methods to manage working memory load in the learning of complex tasks. *Current Directions in Psychological Science*, *29*(4), 394–398. https://doi.org/10.1177/0963721420922183

Powell, S. R., Fuchs, L. S., & Fuchs, D. (2013). Reaching the mountaintop: Addressing the common core standards in mathematics for students with mathematics difficulties. *Learning Disabilities Research and Practice*, *28*(1), 38–48. https://doi.org/10.1111/ldrp.12001

Royer, D., Lane, K. L., Cantwell, E. D., & Messenger, M. (2017). A Systematic review of the evidence base for instructional choice in K–12 settings. *Behavioral Disorders*, *42*(3), 89–107. https://doi.org/10.1177/0198742916688655

Russell, R. (2014). *The impact of using calculators as an accommodation on the math achievement of students with learning disabilities* [Dissertation]. Grand Canyon University.

Scanlon, R. M. (2013). *Improving individualized educational program (IEP) mathematics learning goals for conceptual understanding of order and equivalence of fractions* [PhD executive position paper]. University of Delaware.

Sheppard, M. E., & Wieman, R. (2019). Understanding practice: A pilot to compare mathematics educators' and special educators' use of purposeful questions. *Journal of Human Services: Training, Research, and Practice*, *4*(2). http://scholarworks.sfasu.edu/jhstrp/vol4/iss2/2/

Sheppard, M. E., & Wieman, R. (2020). What do teachers need? Math and special education teacher educators' perceptions of essential teacher knowledge and experience. *The Journal of Mathematical Behavior*, *59*, 100798. https://doi.org/10.1016/j.jmathb.2020.100798

Shult, D. L. (1978). Calculators, computers, and exceptional children. *Journal of Special Education Technology*, *2*(1), 59–65. https://doi.org/10.1177/016264347800200110

Tan, P. (2017). Advancing inclusive mathematics education: strategies and resources for effective IEP practices. *International Journal of Whole Schooling*, *13*(3), 28–38. http://files.eric.ed.gov/fulltext/EJ1153994.pdf

Tan, P., Padilla, A. C., & Lambert, R. (2022). A critical review of educator and disability research in mathematics education: A decade of dehumanizing waves and humanizing wakes. *Review of Educational Research*, *92*(6), 871–910. https://doi.org/10.3102/00346543221081874

Thornton, J., Crim, C., & Hawkins, J. (2009). The impact of an ongoing professional development program on prekindergarten teachers' mathematics practices. *Journal of Early Childhood Teacher Education*, *30*(2), 150–161. https://doi.org/10.1080/10901020902885745

Walsh, J. L. (2001). Getting the "big picture" of IEP goals and state standards. *Teaching Exceptional Children*, *33*(5), 18–26. https://doi.org/10.1177/004005990103300503

Xu, C., Di Lonardo Burr, S. M., & LeFevre, J. (2023). The hierarchical relations among mathematical competencies: From fundamental numeracy to complex mathematical skills. *Canadian Journal of Experimental Psychology*, *77*(4), 284–295. https://doi.org/10.1037/cep0000311