2021

The Impact of Mathematics Curriculum Policy in the United States

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THE IMPACT OF MATHEMATICS CURRICULUM POLICY IN THE UNITED STATES

A MASTER'S THESIS

SUBMITTED TO THE FACULTY

OF BETHEL UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF ARTS

AUGUST 2021

BETHEL UNIVERSITY

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August 2021

APPROVED

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Acknowledgments

I would first like to thank my wife, Mattea, for encouraging me to finish what I started. Despite major renovations on two houses, the birth of our three sons, and moving jobs three times, she was always there reminding me that I could do this. She never complained about the countless 2 A.M. work sessions and always pretended to be interested in the findings of a study from a topic she would never study herself.

Next, I would like to thank my boys, Corbin, Caedmon, and Carver, for being as understanding as young boys can be when their dad keeps falling asleep during story time due to staying up late the night before.

I would also like to thank my advisor, Dr. John Bergeland, for his countless hours of time devoted to guiding me, editing my seemingly endless list of grammatical errors, and encouraging me to keep pressing forward. His patience and understanding when life got crazy was very much appreciated, and his expertise was critical to my success.

Finally, I would like to thank the peers I have worked alongside and the students that have allowed me to be a part of their math journey. You have all shaped me and influenced me in ways I will never be able to fully express. The laughter, the tears, the frustrations, and the celebrations have truly made this extremely difficult profession worth it. You are all mathtastic!

Abstract

The purpose of this literature review was to investigate the effectiveness of common curriculum initiatives in mathematics in the United States. First, the study examined the effects of these initiatives on the rates of students' math anxiety. Second, the study evaluated policies related to curriculum requirements for their effectiveness in improving test scores, closing racial and socioeconomic gaps, and course failure rates. Specific policies were the effectiveness of mandating Algebra 2 and the implementation of Algebra 1 for all eighth-grade students. Third, the study analyzed unique initiatives, such as "Double-Dose Algebra" and math-enhanced elective courses for effectiveness. Findings of current policies were largely either negative or inconclusive, and the results of unique initiatives were mixed.

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CHAPTER I: INTRODUCTION

"Mr. Cook, when am I ever going to use this?" At times, it seems if I had a dollar for every time I've heard this question, I would not have to work a day in my life. Early on in my teaching career, I set out on a mission to have a real-life example of every mathematical concept we studied. "Surely, there is a practical example of every concept I teach," I thought. The further invested I got in that mission, though, the more I realized that some concepts that are required to be taught in high school may not have a direct correlation to real life. Rather, those concepts are simply entry points into more advanced concepts that may or may not apply to my students' lives after high school. Eventually, I burned out on my mission, and it left me wondering whether the curriculum we mathematics instructors are required to teach actually helps our students.

I am now in my eighth year of teaching, the past six of which have been as a high school math teacher. The number of negative, oftentimes visibly repulsed responses I receive when people learn that I'm a math teacher are telling. From my experience, a majority of people leave high school with an extremely negative view of math. Furthermore, many people have expressed how they have forgotten almost all of what they learned during their respective math careers and view their high school math experience as mostly irrelevant to their everyday lives. As well, most have expressed that their time would have been better spent learning more universally applicable concepts such as taxes or budgeting.

These experiences shaped this literature review and guided the investigation of the math curriculum set-up as it exists today. These experiences have driven me to ask crucial questions such as, “Does our system work?” and “What potential improvements have been attempted?”

Chapter I establishes the purpose of this project, defines key terms, and states the thesis. Chapter II is a literature review that investigates the effectiveness of the United States’ current curriculum set up, as well as the efficacy of new initiatives designed to increase performance in high school mathematics. Finally, Chapter III provides an analysis and evaluation of the findings as they relate to the thesis statement, discusses limitations and professional application, and proposes areas for future research.

Rationale

In 1983, a landmark publication entitled “A Nation at Risk: The Imperative for Educational Reform” estimated that only one-third of American 17-year-old students could solve multi-step math problems (Gaertner et al., 2013). Sparked by this report, which asserted that American schools were failing and urged states to raise academic expectations, states have spent decades reforming their policies and curriculum. This reform intensified exponentially following President George W. Bush’s 2001 No Child Left Behind Act, which increased states’ accountability, particularly regarding mathematics curriculum (Kim et al., 2015). The number of states requiring a minimum of 2.5 math credits in high school rose from 12 to 26 from 1987 to 2004, though many of those states did not specify which math courses must be taken (Bozick & Ingels, 2008). As of 2013, 21 states and the District of Columbia had enacted policies that required students to complete a college and career-ready curriculum, which emphasizes completion of advanced math courses typically including Algebra 2 (Aughinbaugh, 2012; Gaertner et al., 2013). To allow for a more rigorous course track in high school, many states have pushed Algebra 1 to eighth grade, eliminating a year of either general math or pre-algebra. Between 1988 and 2007, the proportion of American eighth-grade students enrolled in Algebra 1 doubled (Clotfelter, et al., 2015)

Despite the push for improved math scores on standardized testing, both in Minnesota and nationwide, only 44% and 33% of 8th-grade students were “proficient” or “advanced” on the National Assessment of Educational Progress (NAEP) math test in 2019, respectively. While these percentages may seem shockingly low, scores seem to have increased slightly over the past decade. Stone et al. (2008) cited NAEP data from 2005 in which 37% of 12th-grade students had a “below basic” math proficiency, and only 23% were deemed “proficient.” According to the Minnesota Department of Education website, only 55.5% of students were considered “proficient” on the 2019 MCA assessment (achieved a “meets” or “exceeds”), and the proficiency rate drops to 45.4% when exclusively considering 11th grade performance, the only year high school students take the math MCA. Gaertner et al. (2013) cited data showing only 76% of students graduating high school on time, decreasing to only 60% for Black and Hispanic students. This bleeds through to post-secondary education, where 60% of community college students were enrolled in at least one developmental course.

Internationally, the United States doesn’t fare any better. According to Ruff & Boes (2014), who cited the Program for International Student Assessment (PISA)’s 2009 cross-country comparison of performance among 15 year-olds, “Among 33 industrialized countries, 17 countries had higher average scores than U.S. students, five countries had lower average scores, and 11 countries had scores that were not statistically different from American students” (p. 1). Gottfried et al. (2014) cited the National Science Board’s 2010 publication that stated, “15 year olds in the United states rank...19 out of 26 countries in mathematical literacy" (p. 3). Kim et al. (2015) noted that the average American fifteen-year-old's math scores in 2013 fell well below average when compared to other Organization for Economic Cooperation and Development (OECD) countries. Bozick & Ingels (2008) cited a 2004 study which ranked American 15-year-olds 24th out of 29 countries in problem solving and mathematical literacy.

Policy Shifts in the United States

To improve student scores and raise the United States' ranking, many states have made Algebra 2 completion mandatory for graduation. By mandating Algebra 2, policymakers hoped to close both the racial gap that has historically existed between White students and non-Asian minority students as well as the socioeconomic gap, which correlated with lower level of math course completion (Kim et al., 2015; Stoker et al., 2018). Multiple studies have also indicated that students who complete Algebra 2 are more likely to attend a four-year university and have higher annual income than those who do not take the course (Gaertner et al., 2018; Kim et al., 2015; Stoker et al., 2018). However, there remains significant debate around the impact of selection bias on the data; that is, the notion that students with college aspirations are more likely to take advanced coursework, including Algebra 2. Therefore, Algebra 2 cannot be said to have a causal effect on college enrollment.

Another widespread initiative designed to raise student performance is the movement of Algebra 1 from a required course in ninth grade to eighth grade. Clotfelter et al. (2015) described one benefit of this shift as reducing students' need to "double up" on courses in order to take Calculus by twelfth grade. Another intended byproduct of this shift, like the mandating of Algebra 2, is the closing of racial and socioeconomic gaps. Historically, placing students into "tracks" whereas advanced students take Algebra 1 in eighth grade while low-achieving students take either a general math or pre-algebra course has widened the gap between low- and high-achieving students (Crawley, 2018; Loveless, 2009; Nomi, 2012).

Policy Shifts' Impact

While Minnesota has been a national leader in both of these policy changes, questions remain regarding the benefits as well as the downsides of pushing students who may not be ready for such advanced math courses into such settings. One, does including all students in advanced coursework lower the overall rigor of the courses, and thus hold back the high achievers? Two, do unprepared students actually pass the courses, or do they tend to need to retake them? Three, are these policy changes actually increasing test scores at the state, national, and international levels?

Definition of Terms

For the purpose of this thesis, five specific terms are defined below. Unless specifically defined otherwise, the following terms will use these definitions.

Algebra-for-all

This term describes the movement to require Algebra 1 for all eighth-grade students, regardless of prior math achievement. This movement creates a heterogeneous classroom, where low- and high-performing students learn at the same pace.

Detracking

This is the practice of combining low- and high-achieving students into a heterogeneous classroom. Loveless (2009) added that detracking "reduc[es] the number of subject-area courses offered in a given grade in a given school" (p. 2).

Math Anxiety

This is “an intense fear, nervousness, and dread related to math leading to avoidance of mathematic *[sic]* activities and imped[ing] math learning” (Ruff & Boes, 2014, p. 2). Math anxiety typically prevents students from achieving their full potential.

Math-Enhanced Curriculum

This is the practice of infusing traditional mathematical concepts into elective courses, such as Agriculture classes, thus bringing a math concept to life. For example, a drafting and architecture class can infuse solving proportions into a unit about drawing blueprints.

Socioeconomic Status

This indicates an individual or group’s social status based on factors such as income, employment, and education. While sometimes correlated, race and socioeconomic status do not have to be related.

Tracking

Loveless (2009) defined tracking as “the practice of grouping students into separate classes based on achievement” (p. 1). Tracking produces classes that are homogeneous in nature, where high- and low-achieving students are in separate classes or separate curriculum tracks.

Statement of Research Question

This thesis seeks to answer the following questions: 1) Are the current United States mathematics curriculum policies producing the desired results and optimizing student achievement? and 2) What alternative policies have been attempted in the United States in order to improve student performance in mathematics?

CHAPTER II: LITERATURE REVIEW

The studies in this review analyzed students from fifth grade to post-college, though the vast majority of research focused on middle school and high school. While education in Minnesota was the original focus, it became apparent the review would need to expand nationwide. Simply, there was a lack of research performed in-state with foci matching this review. Despite the expansion, educational policies and initiatives in which Minnesota also participates remained the focus; studies older than 2008 were generally excluded from consideration. However, there were three notable exceptions (Gamoran & Hannigan, 2000; Levine & Zimmerman, 1995; Parr et al., 2006); their research remains relevant today and numerous recent works cite them. Search results were also filtered based on age group; generally, eighth grade was the lower-bound and twelfth grade was the upper-bound. Studies of college students were included only if the study reported effects of high school math or the research could easily also be applied to high school students. Keyword searches in the Bethel University online library catalogue yielded most of the reviewed studies; the remaining studies were located from within the previously reviewed works.

Databases specifically accessed in the Bethel University online library include: EBSCOhost, ERIC, and SAGE Journals Online. The primary keywords used to obtain empirical research included: algebra, detracking, Algebra 2, math, math anxiety, math-enhanced, algebra-for-all, HB 5 Texas, curriculum tracks, and college prep math.

Narrowing Process & Article Selection

The search originally focused on Minnesota math standards. The search was then altered to a general keyword search for “Algebra 2.” This yielded hundreds more results, but many, such as reviews of specific Algebra 2 textbooks, curriculum and programs, were not relevant to this review. However, Texas' House Bill 5 (HB 5) appeared in the results (Porterfield & Hendricks, 2018; Stoker et al., 2018). The main focus of HB 5 was to make Algebra 2 mandatory and was based on a body of work indicating that Algebra 2 leads to higher rates of college completion and job wages. This led to the search for Algebra 2's effect on college and career readiness, where four studies were identified and reviewed (Aughinbaugh, 2012; Gaertner et al., 2013; Kim et al., 2015; Levine & Zimmerman, 1995).

The search was then narrowed to focus solely on the high school curriculum's effect on high school success. Many of these studies analyzed the benefits, or lack thereof, of various course sequences (Bozick & Ingels, 2008; Koon & Davis, 2019; Sciarra, 2010), while one (Son & Senk, 2010) compared The United States' curriculum delivery to South Korea's approach. One recurring theme in these studies was the shift to an "algebra-for-all" model in which Algebra 1 was shifted to eighth grade to provide opportunities for students to take more advanced math in high school. This further narrowed the research to focus on how eighth-grade Algebra 1, a movement at which Minnesota has been at the forefront, was affecting students of all ability levels (Allensworth et al., 2009; Clotfelter et al., 2015; Crawley, 2018; Domina et al., 2014; Gamoran & Hannigan, 2000; Liang, 2012; Loveless, 2008; Loveless, 2009; Nomi, 2012; Sorensen et al., 2018).

Math anxiety's effect on student performance appeared throughout all phases of the research. If students were anxious, the common argument stated, it would be impossible for them to perform at their maximum potential. Since this review's focus was not math anxiety in general, only studies that focused on math anxiety as it relates to currently practiced math initiatives were chosen. This search yielded four results (Brown et al., 2020; Lewis, 2019; Moyer et al., 2018; Ruff & Boes, 2014), one of which (Brown et al., 2020) compared students on an international level.

A theme throughout the research, and one of the main motivations for this review topic, was the effects of high school math initiatives on students with a low level of math comprehension. From experience, many low level students struggle to connect the math they learn with real-world scenarios in which they would use a particular math concept. Results for my posed question ranged from adjustments within the math classroom (Booth et al., 2015; Nomi & Allensworth, 2009; Nomi & Allensworth, 2013) to incorporating math enrichment in Career and Technical Education (CTE) and STEM courses (Gottfried et al., 2014; Parr et al., 2006; Stone et al., 2008).

Chapter II is divided into three main sections: 1) Anxiety and Feelings Toward Math As Potential Barriers to Learning, 2) Analysis of Current Curriculum Shifts, and 3) Non-Traditional Math Initiatives. These sections summarize my thought process to the review's research questions. First, if students' mental states are not conducive to learning, even the best curriculum policy will prove ineffective. This section incorporates more qualitative research than the others and addresses the negative feelings Americans tend to have toward math, even long after they have left the classroom. Second, the bulk of the research focused on the effectiveness of current policies. An old adage states, "If it ain't broke, don't fix it." Here, if current policies are working (or are deemed to be working), then there is no need to change the policies or approaches that are currently in place. Third, there appeared to be room for improvement, so section three addresses the effectiveness of attempted curriculum improvements.

Anxiety and Feelings Toward Math as Potential Barriers to Learning

In the Program for International Student Assessment (PISA)'s 2009 comparison of 15-year old students' math performance, the United States had statistically significantly higher scores than only five out of the thirty-three participating countries, while seventeen countries had significantly higher scores than the United States. Furthermore, in a 2011 national study performed by the National Assessment of Educational Progress (NAEP) merely 82% of fourth-grade students reached only partial mastery of fourth-grade math standards. The lack of improvement, despite decades of educational reform, caused Ruff and Boes (2014) to look toward a common theme throughout time: math anxiety.

Ruff and Boes (2014) defined math anxiety as "an intense fear, nervousness, and dread related to math leading to avoidance of mathematic *[sic]* activities and imped[ing] math learning" (p. 2). The authors also identified three categories of factors that influence a child's math anxiety level: social, cognitive, and academic. Social factors include girls typically having more math anxiety than boys, regardless of ability, and students from low socioeconomic households having a higher rate of math anxiety due to less parental support. Cognitive factors include a range of math-related learning disabilities, or dyscalculia, that can prevent students from having success early on when differentiating numerical magnitude. Academic factors are largely based on the traditional math curriculum that emphasizes memorization, seatwork, and whole class instruction. Such a curriculum can cause students to quickly forget concepts they have learned, thus perpetuating anxiety levels.

While a non-traditional curriculum that focuses on group work and real-life applications has been shown to increase mastery of concepts, Ruff and Boes (2014) acknowledged that curriculum overhauls are not easily implemented and result in overworked teachers becoming even more overwhelmed. Therefore, the authors' attention shifted to the role of professional school counselors (PSCs) and exploring how they could affect change from a psychological perspective. An intervention was performed with three research questions at its core:

> 1. How does math anxiety negatively impact math achievement in fifth-grade students?
> 2. How can PSCs reduce math anxiety and reverse the negative effects on math achievement?
> 3. How can the results from the intervention be used to make improvements in future counseling programs to address math achievement? (p. 5)

To address research Question 1, Ruff and Boes (2014) administered the Math Anxiety Scale for Children (MASC), a 22-item survey that uses a four-point Likert scale, to all 63 fifth-grade students of a suburban public school in Georgia. In addition to the MASC, students were asked five open-ended questions regarding their feelings and perceptions of math. Results from these surveys were compared to each student's score on the winter benchmark exam. Thirteen students did not meet the winter target on one or both math sections, scored highly on the MASC, and obtained parental consent for the study. Of these 13 students, six were female and nine were African-American.

To address research Question 2, the PSC met with the group twice a week for six weeks during lunch. The session topics included:

> identifying and expressing feelings, positive and negative self-talk, changing negative thought patterns, stress reduction and relaxation exercises, self-advocacy-knowing when

> and how to ask for help, goal setting, accepting mistakes as a part of learning, celebrating success, specific math study skills, journaling, [and] self-evaluation. (p. 5)

At the end of the six weeks, students were once again surveyed using the MASC and a list of open-ended questions.

The results showed that six students (46%) had a decrease in MASC score (indicating reduced anxiety levels), four students had an increase, and three students remained the same. Even though MASC scores varied, 10 of the 13 students (84%) increased their scores on the spring MCOMP benchmark exam and 7 of 12 students (58%) improved their scores on the spring MCAP exam (one student was absent for the MCAP).

While the quantitative data was mixed, the qualitative data results were much clearer. Student responses during the initial survey fell into four themes: stress and frustration, negative self-talk and avoidance behaviors, positive attitudes toward math, and positive self-talk and motivated behaviors. Response rates went from 52% to 30%, 18% to 5%, 14% to 30%, and 16% to 35%, respectively.

The authors added two groups of additional questions to the post-survey. In the first group, students were asked, “Imagine you are in math class and you are about to take a test. How do you feel?” (Ruff & Boes, 2014, p. 7). Ninety-two percent of the students expressed stress and frustration. However, when the same situation was described and the students were asked “What do you do?” - 85% of students responded with positive self-talk and motivated behaviors. In the second question group, students were asked to describe how they felt before joining the group. Ninety-two percent of students expressed feelings of stress and frustration.

When asked how they feel about math after participating in the group, 100% of the students described positive feelings and attitudes toward math.

These findings were consistent with the results from the survey given to the classroom teachers as 75% responded “yes” when asked if the small group had had a positive influence on students’ math achievement. Furthermore, 38% of teachers observed increased participation, 38% noted positive attitudes and confidence in students, and 24% noted the students felt less fearful of making mistakes.

One limitation the authors noted several times was the timing of the study. The date for the Criterion Referenced Competency Test was approaching. Success on the CRCT was required in order to move from fifth grade to sixth grade, and students may have been extra anxious about the upcoming test and thus not been able to give the most accurate data possible.

In conclusion, and to explore research Question 3, the authors determined that small group sessions were a beneficial step to reducing students’ math anxiety. Future expansions could include more groups, training faculty to identify and assist students, and helping administration encourage the use of teaching styles that reduces math anxiety.

While most studies on the effects of math curriculum focus on achievement, Moyer et al. (2018) directed focus on students’ attitudes and how those attitudes differed between students who had experienced a reform curriculum and those who had experienced a traditional curriculum in middle school. While traditional curriculum centers around lecture and theory, reformed curriculum uses real-world situations and problem solving to teach concepts. For example, past research found that medical students trained using a problem-based approach (reform curriculum) performed better than those trained in a lecture approach (traditional curriculum) in both factual knowledge and in clinical components. The authors hypothesized that after all students had completed high school, there would be a difference in their attitudes toward math based on which curriculum the students had experienced in middle school.

Moyer et al. (2018) used Di Martino and Zan's three-dimensional model for attitude (TMA) to model student attitudes toward math. The TMA has three dimensions: emotional disposition toward math, perceived competence in math (belief in self), and vision of math (belief about math). The reform curriculum used in the study was the Connected Mathematics Program (CMP), a standards-based curriculum funded by the National Science Foundation that encourages teachers to use a "Launch, Explore, Summarize" method of learning through the increased use of small group inquiry-based learning. The authors posed two research questions:

> (1) Given similar high-school experiences in mathematics, are the TMA profiles of 12th-grade students who had been taught using the CMP curriculum in middle school different from the TMA profiles of 12th-grade students who had been taught using a non-CMP curriculum in middle school? (2) What dimension(s) of attitude contribute to the differences, if any, in their TMA profiles? (p. 119)

For this study, 109 students from 10 different schools were selected by teachers based on in-class effort, willingness to be interviewed (and obtain parental consent), and regular school attendance. These students were a representative sample of the 1000 student LieCal project performed in 2013. Trained teachers interviewed students in ninth grade and again in twelfth grade. By the second round of interviews, only 44 of the original 109 participants were available for study. The other participants had either been chronically absent, transferred schools, or refused to participate. Of the remaining 44 students, 18 had a CMP experience in middle school, whereas 26 had a non-CMP experience. There was no significant difference in achievement between students at the time of the second interview, and enrollment in remedial, standard, and advanced courses had similar proportions. Sixty-one percent of participants were African American, 25% were White, 11% were Hispanic, and 2% were Asian. Almost two-thirds of the participants were female, and 68% were eligible for free or reduced lunch.

Through interviews, Moyer et al. (2018) found “the proportion of CMP students who had a positive Emotional Disposition was not significantly greater than the proportion of non-CMP students who did” (p. 126). Regarding Perceived Competence, analysis showed that CMP students did not feel more or less proficient in mathematics than their non-CMP peers. The third dimension, students’ vision of math, did show statistical differences. Students tended to align with the curriculum style they had experienced in middle school; CMP students tended to be more dependent upon themselves or their peers for learning concepts rather than the teacher. Conversely, non-CMP students tended to depend on teacher assistance and to believe learning was dependent on their teacher.

Lewis (2019) studied 221 University of South Alabama students’ math anxiety, test anxiety, and communication apprehension as they pertain to a math classroom following a standards-based grading approach. More specifically, Lewis wanted to determine whether student anxieties and apprehensions decreased over the semester and how those feelings affected the number of times a student used the option to do an office-hours reassessment to obtain a higher score.

Students who were enrolled in one of 13 sections of sophomore-level courses taught by four different professors were surveyed at the beginning and end of the semester regarding their anxieties and apprehensions. Most questions were quantitative and used a Likert scale, but some questions were open-ended in nature. Two professors offered written reassessments and two offered oral reassessments in which students would describe how to do a problem on the board. This allowed Lewis (2019) to study whether students with anxieties and apprehensions responded better to written or oral reassessments.

Students' math anxiety and communication apprehension remained steady throughout the semester, while their test anxiety rose significantly. Furthermore, of the 70% of students who attempted at least one office hour reassessment, written reassessments were preferred over oral reassessments. Lewis (2019) concluded that the standards-based grading approach was beneficial for students with high test anxiety but low communication apprehension since those students expressed relief due to the retake opportunity. Conversely, standards-based grading does not appear to have been beneficial for those who have high levels of both text anxiety and communication apprehension, since those students seem bound to continue to feel trapped in their anxiety. For those students, the author suggested more and/or different approaches to help them feel more at ease. Different approaches suggested included requiring students to meet once or twice at the beginning of the course in a low-stress environment to "break the ice," allowing alternate evidence of mastery, and conducting office hours in a less intimidating environment (e.g., the library, a coffee shop).

While many factors influence math performance, Brown et al. (2020) studied the effect of math anxiety on students. More specifically, Brown et al. (2020) wondered whether math anxiety was just a problem in the United States, or whether students from other countries had the same level of negative feelings toward the subject. Here, math anxiety is defined as “the unpleasant state of mind characterized by anxiety, fear and concern when facing situations where they are required to perform in mathematics” (p. 127). Since higher levels of anxiety tend to prevent individuals from reaching their maximum potential, reducing levels of math anxiety in American students would likely have a positive impact on scores.

The authors compared students from both the United States and Colombia who were enrolled in a post-secondary engineering program. In 2015, Colombian students ranked 61st out of 70 countries on the PISA test, whereas American students ranked 40th. Brown et al. (2020) used the Abbreviated Math Anxiety Scale (AMAS) to answer the following research questions:

1) What is the difference in learning maths anxiety and maths evaluation anxiety for engineering students at two universities with different cultural backgrounds?
2) What is the difference in learning maths anxiety and maths evaluation anxiety for engineering students by gender?
3) Is there an interaction effect between cultural groups and gender for engineering students at two universities with different cultural backgrounds? (p. 130)

Twenty engineering students from a university in the south-eastern United States (16 males, 4 females) and 88 engineering students (58 males, 30 females) from a university in Colombia were selected for the study. The AMAS used consisted of nine questions divided into two subgroups, Learning Math Anxiety (LMA) and Math Evaluation Anxiety (MEA). The five LMA questions measured anxiety when doing math during either class time or study time, while the four MEA questions measured anxiety during math assessments. For all questions, the scale ranged from 1 (low anxiety) to 5 (high anxiety).

Two-way factorial ANOVAs were conducted for both LMA scores and MEA scores. There was a statistically significant difference in LMA scores; the American university averaged 1.74 while the Colombian university averaged 2.61. The authors noted that this could have been due to Colombian instruction, as it remained focused on the expository while technological advances have required students to shift their method of learning. Comparisons of LMA in males and females yielded no significant results, and therefore there was also no significant interaction effect between culture and gender. The MEA segment results were not statistically significant, although they were higher than LMA scores by 1.14 overall.

In conclusion, Brown et al. (2020) noted that cultural background affected math anxiety, although only in LMA. More strikingly, though, was the increase in anxiety between LMA and MEA. According to the authors, “This makes one wonder if the way of managing the evaluation in the different mathematical subjects is being approached in the correct way, due to the high values of anxiety that they are provoking in the students. Therefore, they should be reviewed to try to reduce this handicap and thus generate more self-confidence in them to guarantee that these tests are carried out in opitmal *[sic]* conditions” (p. 134).

Analysis of Current Curriculum Shifts

The United States has trailed South Korea for many years in mathematics education, at least as it pertains to standardized testing, so Son and Senk (2010) set out to compare the similarities and differences between the United States’ *Everyday Math* textbook and South Korea’s *7th Korean* mathematics textbook. The researchers specifically looked at the foundational concept of multiplication and division of fractions to gain insight into each country's approach and instruction of a key math concept. The authors examined both the quantity and quality of problems as well as the expectations each text set upon the students regarding building conceptual understanding and procedural fluency. In order to do this, Son and a doctoral student, both of whom are fluent in English and Korean, independently reviewed the materials to determine how many lessons and problems were devoted to each concept. They then coded problems based on factors such as the number of steps required, computational strategies used/taught, and level of difficulty. After comparing scoring rubrics, themes, and patterns, Senk reviewed the results.

The authors discovered that the South Korean textbook has a higher portion of multi-step problems, (18% of multiplication problems compared to 1%, for example) and also requires a greater variety of response types than the United States textbook. Additionally, while the United States and South Korea place similar value on multiplying fractions (nearly identical amount of practice problems and lessons spent on the topic), South Korea places a significantly higher emphasis on dividing fractions than the United States. This is evidenced in the *7th Korean* textbook that contained five times more lessons and eight times more problems on dividing fractions. Son and Senk (2010) concluded that the lack of multi-step problems presented in the United States textbook appears to be one reason American students do not do as well compared to their South Korean peers on standardized tests.

Aughinbaugh (2012) built upon past work in an effort to answer the question "Does a more advanced math curriculum result in higher rates of college enrollment?" The author used the data from the National Longitudinal Survey of Youth 1997 (NLSY97) which also collected data from the students' high school transcripts. The author then narrowed down the data to only include households that had multiple data points within the NLSY97. That way, factors such as household income and race could have potentially been controlled. For this study, Aughinbaugh (2012) required college attendance to have been decided by age 21. Before starting the study, the author acknowledged a common concern when considering how higher math curriculum influences college enrollment rates. Namely, do students enrolled in higher math go to college at higher rates due to motivation or due to the curriculum? To add clarity to the motivation levels of each student, the author also analyzed data from the Armed Services Vocational Aptitude Battery (ASVAB).

Before controlling for household effect, there were 5,233 students who met the criteria for the study. These students were sorted into four curriculum groups: "No or Low Math," "Mid-Academic 1," "Mid-Academic 2," and "Advanced" (p. 865). At this level, the results clearly indicated that the rate of college enrollment increased with each level of math curriculum. Aughinbaugh (2012) pointed out that students in more rigorous math courses also had higher rates of foreign language and advanced science enrollment, as well as higher GPAs.

About 32 percent of the students in the sample qualified to be included in the household fixed effect model (n=1673). Interestingly, 55 percent of the students were in different curriculum groups than the other members of their household which provided a natural control for factors previously mentioned such as household income.

Aughinbaugh (2012) found that students in the advanced curriculum group (highest course being Algebra 2, Precalculus, Trigonometry, or Calculus) were about 17 percent more likely to attend college and about 20 percent more likely to begin a 4-year program by age 21 than students who only took Algebra 1 or Geometry. Furthermore, students in the advanced curriculum group were more than 150 percent more likely to start college at a 4-year college than students in the mid-academic curriculum group. The author noted that these findings were similar to those of other studies.

Levine & Zimmerman (1995) studied the effects of taking more high school math and science classes on wages, rate of entering a technical job, and choice of technical college major. Specifically, the authors addressed both the wage gap between men and women and the lower rate of women in technical fields. At the time of this study, little research had been conducted regarding the benefits of high school curriculum as it pertained to labor market outcomes. It was the authors' intention to fill that gap.

Two sources of data, the National Longitudinal Survey of Youth (NLSY) and the 1980 cohort of the High School and Beyond (HSB), were used in this study. These studies included an array of useful information, such as data on participants' wages, occupations, educational experiences, families, and high school transcripts. After eliminating subjects that had dropped out of high school and who were not employed full time for the full year in 1990, the authors arrived at sample sizes for the NLSY and HSB of 3,920 and 5,493, respectively.

Concerning wages, the NLSY showed that women in the study made 74% of what men made, and the HSB showed 82% of men's earnings. The authors also found that men take two percent more math and science classes than women on the NLSY and a half a year more math and science than women on the HSB. There was no difference in reading and vocabulary. This led Levine & Zimmerman (1995) to perform regression analysis to learn more. While most of the results were inconsistent, there was one noteworthy finding. Women who were college graduates and took additional math classes in high school consistently appeared to have increased wages. Science did not appear to have an influence on wages in any groups studied.

Next, the authors studied the effect of additional math and science courses' on men and women entering a technical profession. Results from the NLSY showed that women who complete more math classes are statistically more likely to enter more technical occupations. The HSB showed that both men and women benefited from additional math and science courses, but when men and women were separated by educational attainment this finding became statistically insignificant.

Levine & Zimmerman (1995) then analyzed the effects of additional math and science on college graduation. They found that in the NLSY, men who took more math or science courses were more likely to attend college; however, men were not statistically more likely to graduate from college. Additional courses did not affect women's college graduation rates in the NLSY. The HSB, however, showed that women who took more math classes were more likely to graduate from college. In this case, science classes had no impact.

Upon further research, the authors discovered that both data sources had a positive correlation between additional math courses and women who graduated from college and entered fields not traditionally dominated by women. In the NLSY, a two percent increase in math classes led to a three percent increase in the probability of women entering a technical field and an almost three percent decrease in choosing a traditionally female course of study. The HSB had similar findings, there was a slightly bigger decrease (four percent) in women who chose a traditional course of study. Science had a weak correlation as well once the authors controlled for those entering nursing the correlation ceased.

In conclusion, the authors found that women who took additional math courses resulted in increased wages for those who graduated from college. Women also were more likely to enter technical jobs and graduate from college with a technical degree. Results for men were inconsistent, and science courses did not appear to have played a significant role.

As for educational policy and initiatives over the past thirty years, much emphasis has been placed on helping students be “college and career ready.” In those circles, it has been implicitly expressed that college readiness requires the same things as career readiness.

Gaertner et al. (2013) investigated this implicit assumption and applied it to studying the amount of influence Algebra 2 has on both aspects of the assumption. While numerous studies claim Algebra 2 completion leads to higher college success rates, the authors questioned whether this was due to the fact that college-ready students typically take higher level courses. In other words, is it the case that Algebra 2 causes students to be college-bound more often, or that being college-bound causes students to take Algebra 2? Ultimately, the main question was, “Do mathematics course-taking patterns in high school influence college and career outcomes in the same ways?” (p. 149).

The authors used two nationwide studies, the National Education Longitudinal Study of 1988 (NELS) and the Educational Longitudinal Study of 2002 (ELS). The NELS followed up with participants in 10th grade, 12th grade, two years post-completion, and eight years post-completion, while the ELS followed a similar timeline. This parallel setup allowed for analysis across two data groups and increased confidence in the results. Gaertner et al. (2013) also accounted for two additional factors that could potentially sway the data: (1) the unemployment rate in the student’s zip code during their 10th-grade year, and (2) whether or not a student was 16 years old or older during their 10th-grade year. Both of these factors were significant because unemployment and extended work availability under Child Labor Laws may have caused students to choose work over more difficult classes. It is important to note that the national unemployment rate was very similar in both the NELS and the ELS.

Upon analysis, Algebra 2 participation rates saw a huge increase from the NELS to the ELS (45% to 70%). This increase is important for there is a possibility that the ELS data may have been watered down in some way. Regardless, the analysis resulted in several conclusions. One, students who completed Algebra 2 tended to have a higher college GPA; the NELS data showed a 0.83 increase ($p<0.001$). Two, Algebra 2 completion has a strong correlation to college retention and graduation rates. For example, college graduation rates for NELS participants were 29.4% higher for those who completed Algebra 2. Three, first and second year retention rates in both NELS and ELS were significantly higher for those who had completed Algebra 2, although ELS had lower increases. This is likely due to the increase in Algebra 2 participation rates as previously discussed.

As it pertains to earnings, the data was far less clear. In fact, Algebra 2 appears to have had a negative impact on initial earnings, yet the long-term earnings tended to be higher for those who completed the course. However, the authors cautioned these higher earnings were unlikely to be solely attributed to Algebra 2 completion. Additionally, the sharper rise in salary was likely due to degree attainment and white collar vs. blue collar work environments. NELS participants who completed Algebra 2 were six percent more likely to retain their occupational prestige. There was also a slightly higher chance of career advancement for Algebra 2 students in the NELS, but this slight increase was not replicated in the ELS data.

Gaertner et al. (2013) concluded that "college and career readiness are not the same thing" (p. 161). Also, Algebra 2 did not influence them in the same ways, "Algebra 2 completion is generally a stronger determinant of college success than career success" (p. 161). Furthermore, "[O]ur focus was on career outcomes for students who do not enroll in college and instead enter the workforce immediately following high school. For that group, finishing Algebra 2 does not confer many measurable benefits" (p. 161).

While there have been several studies performed to examine the correlation between advanced math placement and college success, many of them have not taken into account biases that likely overestimate the impacts of advanced math. Kim et al. (2015) hoped to use different statistical analysis methods to avoid such biases as they explored whether advanced math completion in high school, as well as specific higher level courses, resulted in increased rates of baccalaureate degree attainment. More specifically, their study centered around Algebra 2 and focused on two questions: "1. Does completing Algebra 2 in high school affect whether, and if so the type of college that a student attends? 2. Does completing Algebra 2 in high school affect whether, and if so the type of degree a student attains?" (p. 635).

Kim et al. (2015) accessed longitudinal data from Florida's student unit record on the Educational Data Warehouse. Originally, this data set encompassed 758,456 students in grades 7-12 enrolled during the 1995-1996 school year. The authors pared down the data to 615,185 students with 12th grade records to address research question 1. They further restricted the data for the second research question to only those who had enrolled in college, with N = 427,845. Kim et al. (2015) controlled for a number of factors, including race and ethnicity, gender, free/reduced lunch status, language spoken at home, number of AP and IB credits earned, SAT scores, GPA, and school district mission differences. They determined the variable of interest, level of mathematics completed in high school, to be binary, with Algebra 2 completion as the threshold.

To explore the effects of different variables, Kim et al. (2015) first used a multinomial logistic regression (MNL). However, they found a potentially significant area of bias and therefore employed an instrumental variable (IV) approach. The instrumental variable used was the unemployment rate in the county in which the student during ninth grade. This IV even allowed the authors to separate unemployment rate from family income considerations as they examined the effects of both on Algebra 2 completion. Although unemployment rates remained fairly steady (3.8% in 2001 was the lowest, while 5.7% in 2003 was the highest), Kim et al. (2015) included the rates from both the students' freshman and senior years to remove any chance of potential correlation. Unemployment rates were chosen because:

> Local labor market conditions when students are in high school may affect their college preparation decisions and in so doing affect college attendance and completion. For example, in a weak (strong) local labor market students may allocate less (more) time to work and more (less) time to study by increasing (decreasing) the quantity or difficulty of the courses that they take in high school. (p. 642)

When analyzing results of the study, Kim et al. (2015) compared their initial "naive" MNL regression with their IV model in order to determine the amount of potential bias that may reside. Regarding whether or not Algebra 2 completion affected college enrollment (Question 1), both models agreed that Algebra 2 completion increases the probability of four-year college enrollment by approximately 20.6%. However, the MNL regression revealed bias when considering two-year college enrollment. While the MNL regression showed an 18.5% probability of attending a two-year college (2.1% lower than students who did not complete Algebra 2), the IV model indicated the increased probability was actually 27.6%. The models also found that overall, students who do not complete Algebra 2 had a 48% lower chance of enrolling in post-secondary education. To summarize the findings to the first question, Kim et al. (2015) stated, "The results indicate that for statistically similar students, completing Algebra 2 in high school dramatically increases their chances of college attendance in general and is also related to the type of postsecondary institutions they attend" (p. 652).

The results for question two contained apparent bias in the naive MNL regression. The MNL regression indicated that Algebra 2 completion increased associate's degree completion by 2.1% and bachelor's degree completion by 5.6%. It also showed that students who complete Algebra 2 are about eight percent less likely to obtain a postsecondary degree than their non-Algebra 2 peers. The IV model results, on the other hand, revealed the increase in bachelor's degree completion was not statistically significant when they took student self-selection into consideration. Similarly, the increase in associate's degree completion was also statistically insignificant. This divergence in results between models confirms that many previous studies likely have had selection bias that caused the results to overestimate the importance of Algebra 2 completion. Kim et al. (2015) responded to the second question as follows:

> ...our results suggest that completing Algebra 2 in high school has a positive effect on postsecondary education attainment of any kind, but the effect is only significant for

> two-year attendance, rather than four-year college attendance. Conditioning on college attendance, we found no Algebra 2 effect on degree attainment. (p. 652)

For future studies, Kim et al. (2015) encouraged consideration of more specific class' influence on their research questions. The authors acknowledged they may have presented a bias by treating those who stopped after completing Algebra 2 and those who went beyond Algebra 2 the same.

While No Child Left Behind and Common Core standards have pushed states to require Algebra 2, some states have chosen alternative requirements. For example, Florida eliminated Algebra 2 as a graduation requirement beginning in the 2010-2011 school year. In the 2017-2018 school year, Virginia increased emphasis on internships and work experience. Texas' 2013 House Bill (HB) 5 maintained the requirement to take four math courses, but eliminated an Algebra 2 completion requirement that had been in effect for seven years. HB 5 was controversial, supporters praised the ability for non-college bound students to focus on classes more applicable to interests and career aspirations, which may have lowered failure and dropout rates. In place of Algebra 2, students could can choose from a long list of alternative courses, including, but not limited to Accounting II, Algebraic Reasoning, Digital Electronics, Financial Mathematics, Robotics II, Statistics and Business Decision Making, and Mathematical Applications in Agriculture, Food, and Natural Resources. Critics of the bill argued that all students should be challenged in preparation for success in the workforce, something that might not occur without the Algebra 2 requirement. Additionally, critics argued that lack of Algebra 2 may further reduce enrollment of minority students in postsecondary education programs.

Stoker et al. (2018) conducted a study to compare pre- and post-HB 5 student performance data. Specifically, they answered the following questions:

1. How are districts implementing the curriculum requirements for graduation under HB 5?

2. What is the trend in Algebra 2 completion rates by the end of grade 11 for students who entered grade 9 during the 2007/08–2014/15 academic years?

    • Does the trend differ by student and district characteristics?

3. What is the trend in Algebra 2 failure rates by the end of grade 11 for students who entered grade 9 during the 2007/08–2014/15 academic years?

    • Does the trend differ by student and district characteristics?

4. What math courses did students take who entered grade 9 during the 2007/08–2014/15 academic years but who did not take Algebra 2 as their third math course? (p. 3)

For question one, the authors were given survey results from a district-level online survey administered in spring 2015. The remaining questions used longitudinal data from the Public Education Information Management System (PEIMS), statewide assessment files from 2011/2012-2014/2015, and Texas Academic Performance Report (TAPR) from 2007/2008-2015/2016. The district-level survey yielded 890 district responses (81% of all Texas public school districts). The responses accurately reflected statewide student demographic numbers for English language learners, minority students, economically disadvantaged, and students in special education programs. However, the data sample underrepresented charter schools and districts with fewer than 500 students. An additional study performed by the authors followed eight cohorts of ninth-grade students who had the chance to complete Algebra 2 by the end of their eleventh-grade year. This study only analyzed those students who completed three consecutive years of education in a Texas public high school.

Stoker et al. (2018) calculated descriptive statistics from the survey results and found 94% of the districts were encouraging their students to attain the Distinguished Level of Achievement (DLA), an advanced course track that includes Algebra 2. Forty-nine percent of districts stated the coursework for the DLA was embedded into their requirements for graduation, and 37% of districts reported still requiring Algebra 2 completion for graduation. Additionally, few districts indicated a plan to begin offering the alternative courses mentioned by the state. Only 30% of districts planned to offer Algebraic Reasoning, while 44% planned to offer Statistics.

Although a gap remained, Algebra 2 completion rates among demographics remained steady during the transition to HB5 requirements. This consistent trend also followed students who were economically disadvantaged, with 72% of economically disadvantaged students completing Algebra 2 compared to 84% of non-economically disadvantaged students. Similarly, district-level trends remained steady, with urban districts generally having higher completion rates than rural and charter schools.

Failure rates also stayed consistent at 14-17%. Black and Hispanic students failed at a higher rate than White and Asian students, and districts with a high ethnic/minority population saw higher failure rates as well. Additionally, those who scored low on the STAAR exam in eighth grade had an Algebra 2 failure rate almost six times higher than those who had performed well. However, none of these findings appeared to be affected by HB5 implementation.

Finally, Stoker et al. (2018) researched which alternative courses students took if they had not completed Algebra 2 by their eleventh-grade year. Most students instead enrolled in Mathematical Models, a pre-Algebra 2 course. There was also a slight increase in students taking Geometry their junior year and not continuing on to additional higher math courses. The authors acknowledged that following the cohorts for four years rather than three may yield slightly different results. Additionally, the authors encouraged further studies to be done now that more time has passed since HB5 was enacted. During this time, schools may have changed their course offerings, and more students may now be choosing alternative course paths.

The class of 2013 had a record graduation rate of 88% in Texas public schools. However, only 33.9% of students during that time were deemed “college-ready” on the SAT, nearly nine percent lower than the national average. Furthermore, only 56% of the class of 2013 were considered ready for college in both math and English. Their predecessors did not fare much better; only 60% of students from the classes of 2011 and 2012 enrolled in post-secondary education, and over 30% of those students were placed in college remediation courses.

In 2013, the Texas legislature passed House Bill 5 (HB 5) which decreased standardized testing and changed graduation requirements. Under HB 5, students are required to complete an endorsement in addition to the foundational courses. Endorsements in STEM, business and industry, public services, arts and humanities, or multidisciplinary studies are offered with the intention of giving students the opportunity to become more career focused. HB 5 also allowed local districts the freedom to develop partnerships with local industries to provide students the opportunity to achieve industry-recognized credentials. Career and Technical Education (CTE) courses were also encouraged for students who wished to pursue vocational fields in place of advanced math coursework.

Porterfield and Hendricks (2018) studied the change in district program enrollment pre- and post-implementation of HB 5. High schools selected ranged in size from 2A (small) to 6A (large) and were classified as either property-wealthy (Chapter 41) or property-poor (Chapter 42), as defined by the Texas Education Code. The authors created separate Chapter 41 and Chapter 42 samples that included 61 6A schools, 46 4A schools, and 45 2A schools (n = 152).

In this study, the authors posed the following questions and included their null hypothesis to each question:

R1: What is the difference in advanced course completion for Texas Chapter 41 high schools before and after HB 5?

$H_0$1: There is no difference in advanced course completion for Texas Chapter 41 high schools before and after House Bill 5.

R2: What is the difference in advanced course completion for Texas Chapter 42 high schools before and after HB 5?

$H_0$2: There is no difference in advanced course completion for Texas Chapter 42 high schools before and after House Bill 5.

R3: What is the difference in CTE coherent sequence completion for Texas Chapter 41 high schools before and after HB 5?

$H_0$3: There is no difference in CTE coherent sequence completion for Texas Chapter 41 high schools before and after House Bill 5.

R4: What is the difference in CTE coherent sequence completion for Texas Chapter

42 high schools before and after HB 5?

$H_0$4: There is no difference in CTE coherent sequence completion for Texas Chapter 42 high schools before and after House Bill 5. (p. 7)

Porterfield and Hendricks (2018) found that advanced course completion in Chapter 41 schools increased slightly from 54.1% to 54.3%, but this was statistically insignificant. Chapter 42 schools experienced a small to medium practical significance (d = .29 Cohen's effect size value) result, as the advanced course completion rate increased from 48.7% to 51.9%. Therefore, the authors' second null hypothesis was rejected. Research questions three and four, like question one, saw slight but statistically insignificant increases. CTE course completion in Chapter 41 schools increased from 53.4%to 55.3%. Chapter 42 schools' CTE course completion increased from 56% to 59%.

The authors concluded that while all results saw an increase in completion rates, only one null hypothesis was rejected. Therefore, no conclusions can be made about HB 5's impact. The overall inconclusiveness in this study prompted the authors to suggest that another study should be done after the class of 2018 (the first class to have HB 5 influence over their entire high school career) graduates.

Bozick & Ingels (2008) used the Education Longitudinal Study of 2002 (ELS: 2002) to study how various demographics and school structures affected student coursetaking in mathematics. The ELS: 2002 is a nationally representative study that followed students enrolled in 10th grade during the 2001-2002 school year. Out of the 14,710 participants in the ELS: 2002, the authors used data from 9,460 students who met the study criteria to answer the following research questions:

1) How much does mathematics achievement change during the last two years of high school and are these changes related to student background and school characteristics?
2) What are the most common mathematics course sequences taken by students in the 11th and 12th grades and are these sequences related to student background and school characteristics?
3) What mathematics course sequences are associated most closely with mathematics achievement? (p. 4)

A follow-up test was conducted when the students were finishing their 12th-grade year, as well as in the spring of 2006 (two years post-graduation). Transcript data was also collected. Due to the authors' criteria that participants must have not dropped out, transferred schools, or switched to homeschooling, Bozick & Ingels (2008) acknowledged that their sample included a higher proportion of White students who lived with both parents and earned Bachelor's degrees or higher. They noted this was unlikely to produce invalid conclusions, but it needed to be taken into account.

Upon analysis of both number-right scores and levels of proficiency, Bozick & Ingels (2008) found that, in regard to RQ1, students on average increased their number right by about five questions, and all subgroups showed gains. Students attending Catholic schools showed the biggest improvement. As sophomores, 53% of students were at a Level 3 proficiency and 25% were at Level 4 (5 being the highest). By the end of high school, those percentages had shifted to 65% and 38%, respectively. The authors found no significant difference between boys and girls on either the 10th-grade or 12th-grade assessments. Race/ethnicity, however, did show some differences. While there were no differences in gains over the years, White and Asian students tended to enter 10th grade with a higher proficiency level than their Black and Hispanic peers. Eighty-three percent of Asian students and 82% of White students entered 10th grade with a Level 2 proficiency, while only 54% of Hispanic students and 46% of Black students entered at a Level 2 proficiency. By the end of high school, 52% of Asian students and 45% of White students had achieved a Level 4 proficiency, while only 20% of Hispanic students and 12% of Black students had achieved the same level. Students with high socioeconomic status (SES) made higher gains in more advanced math than their low SES peers, whose gains occurred mainly in intermediate-level skills. Finally, students who attended Catholic schools, students with college degree aspirations, and students who lived with both parents all tended to end high school with a higher proficiency level than their peers, though there were no significant differences in gains across groups. This indicated that those students weren't necessarily improving more, but rather they were being exposed to more advanced concepts earlier in their math careers.

This finding led into the authors' findings for RQ2 in which course sequences were analyzed. Bozick & Ingels (2008) identified 16 possible courses that a student could take, ranging from "No Mathematics" to "Advanced Placement/International Baccalaureate Calculus" (p. 16). Students' course sequences over their final two years of high school were compiled, and of the 256 possible combinations, only six applied to more than 5% of students and nine contained the minimal number of students to be considered in the analysis. The most common course sequence was the Algebra 2-No Mathematics, meaning students took no math their senior year; in fact, 12.7% of students followed that path. The most common sequence where students took two math courses was Algebra 2- Precalculus at 6.8%. Demographic results were largely consistent with the findings from RQ1. There were no significant differences between boys and girls. Asian and White students were approximately 10% more likely to take Precalculus than Black and Hispanic students. Students living with both parents were seven percent more likely to take Precalculus than those from single-parent homes. Students seeking a college degree were more likely to take advanced math courses. Catholic school students were about 10% more likely to take Precalculus than their public school peers. The one significant finding pertained to high SES students, where almost 30% took a sequence involving Precalculus as compared to 11% of low SES students.

Lastly, RQ3 data showed a clear correlation between taking two math courses and higher gains in correct answers. For example, students in the Algebra 2- Precalculus sequence gained 6.9 correct answers on average, while students who chose Algebra 2-No Mathematics only improved by an average of 4 answers. Students in the most advanced sequences, such as Precalculus-AP/IB Calculus had gains with effect sizes of nearly one standard deviation. Students in the No Mathematics-No Mathematics track showed the least improvement with a change of just 1.4 correct answers. Proficiency gains, though, revealed some interesting insights. Improvements in proficiency level 1 (the lowest level) were highest with the Geometry-Geometry/No Mathematics track. Since much of the content in these courses are basic and foundational in nature, it makes sense that these students ``topped out” at a foundational level. Improvements at levels 2 and 3, the intermediate proficiency levels, were greatest in the Geometry-Algebra 2 students. Gains at this level in the advanced sequences was limited, mostly due to the fact that most students in those tracks started at a Level 2 or 3 rather than growing into it. Perhaps unsurprisingly, proficiency Levels 4 and 5 saw the most growth from the Algebra 2-Precalculus and Pre-Calculus-AP/IB Calculus sequences, respectively. One interesting note regarding Level 5 proficiency, however, was that there was clearly room for growth as 71% of Pre-Calculus-AP/IB Calculus students were not yet at that level.

In conclusion, Bozick & Ingels (2008) noted that while students seem to enter with a grasp of lower-level skills, only 4% of students leave high school with the highest level of proficiency. Furthermore, the data showed that while students in advanced math sequences learn more advanced skills, success in the final years of high school has more to do with previous math experience and background characteristics. While students who are high SES, White, Asian, from two-parent households, and/or attend private schools are more likely to take advanced math courses, it was apparent that this was due more to opportunities earlier in life rather than influences in high school. “[T]his study shows that despite the significant relationship between mathematics course taking sequences and achievement gain, a greater amount of the variation in learning at the end of high school is explained by factors that precede enrollment in these coursetaking sequences” (p. 34).

Based on the 2018 Mississippi Department of Education finding that fewer than one in five Mississippi eleventh-grade students were deemed ready for college math courses in 2016-2017, Koon and Davis (2019) set out to determine if there was a correlation between demographics (e.g., race, gender, national school lunch program), elementary math scores on Mississippi state standardized testing, and 6-11 course sequencing as it related to the likelihood of a student being deemed “college-ready” (defined as scoring at least a 22 on the ACT). The study analyzed data from 27,680 eleventh-grade public school students across the state.

First, the authors examined the different combinations of courses that students had taken while attending grades 6-11 and clustered them based on similarity. In Mississippi, students are required to take Algebra 1, but they then have the freedom to select the sequencing of the remaining courses needed for graduation. Koon and Davis (2019) identified 3,404 different course sequences throughout grades 6-11 from the data, which they grouped into six distinct clusters. Next, they entered demographic data into the clusters to identify possible themes, such as whether or not low-income students tend to follow a certain course track. Last, they added and evaluated students' fifth-grade test scores.

Koon and Davis (2019) determined that more girls than boys and more white children than black children followed the course track with the highest achievement, wherein students take Algebra 1 in eighth grade followed by Geometry, Algebra 2, and Algebra 3. On the other hand, students who took eighth-grade pre-Algebra followed by both Algebra 1 and Compensatory Math 1 in the ninth-grade course track performed the lowest. More black students than white students and more students who were eligible for the national school lunch program than those who were not eligible followed this course track.

Interestingly, the selected course track did not have a sizable impact on college readiness. Rather, a combination of fifth-grade test scores (the main influence) and race predicted 11th-grade college readiness with 88% accuracy. Based on these findings, the authors encouraged further research in regard to increasing the effectiveness of elementary school math instruction.

Instead of considering Algebra 2 as a predictor variable for future events, such as college attendance and completion, Sciarra (2010) viewed it as an outcome variable. In other words, the author set out to find other factors that either increase or decrease the likelihood of a student taking Algebra 2 and advanced math. Specifically, Sciarra (2010) posed the following research questions:

> Are there certain predictors that allow us to account for the variance in separating those who complete more intensive math from those who do not? What background variables account for the greatest significance? Is intensive math course-taking simply a function of cognitive ability (math standardized scores) and/or achievement (GPA)? What happens to the significance of demographic factors once academic factors are added to the equation? (p. 197)

To answer these questions, Sciarra (2010) analyzed data from the 2002-2004 Educational Longitudinal Study (ELS). This nationally representative study looked at 10th-grade students in 2002 and followed up in 2004, including transcript data from all four years of high school. Of the 15,362 initial participants, 11,909 met the criteria for this study. Weighting adjusted for potential biases and unequal probabilities. Sciarra (2010) used eight background variables (i.e., gender, race, SES, family composition, student expectations, parent postsecondary aspirations, ESL, and bilingual class) as the independent variable and academic course-taking (i.e., Algebra 2 or less and more than Algebra 2) as the dependent variable.

Upon first analysis, gender, ESL, and family composition had no significant effect on the dependent variable and were therefore dropped from further analysis. In the next analysis, the author included two academic variables, 10th-grade achievement scores and GPA, and the results indicated that all five remaining background variables had a significant influence on the dependent variable. Students in the highest SES quartile were 3 times, 2.5 times, and 2 times less likely to be in the "Algebra 2 or less" category than students in the lowest quartile, 2nd quartile, and 3rd quartile, respectively. If a student in the lowest quartile was performing well academically, however, their chances of going beyond Algebra 2 increased by almost 50%. Racially, "Native Americans were almost 4 times more likely, African Americans 1.5 times more likely, and Latinos twice as likely to complete a course in Algebra 2 or less than Whites" (p. 201). Overall, the independent background variables accounted for 27% of the influence on the dependent variable when the independent academic variables were not factored in and 46% when GPA and achievement scores were factored into the analysis. In fact, all impacts of background variables decreased in significance when the authors considered prior math achievement, indicating that prior math success has more influence than background on predicting future math success. However, background variables still have enough influence that they deserve consideration when discussing how policy changes affect student performance.

Tracking, the practice of assigning students to curriculum paths based on perceived ability level, used to be common. However, in the 1990s, an anti-tracking movement argued that tracking was discriminatory and created a wider gap between high and low achievers. Even so, not all schools were detracked. Previous studies found tracking to be less common in schools that served students typically hurt by the practice, such as low-income and minority students. Loveless (2009) updated research on the topic; the focus was on middle school students in Massachusetts during the 2008-2009 school year. This study's research questions were 1) "How has tracking changed since the 1990s?" 2) "Why have some schools detracked, while others resist the reform?" and 3) "Are there differences in the number of higher-achieving students associated with tracked and untracked schools? In other words, do tracked and untracked schools produce similar percentages of students reaching the “advanced” level on MCAS?” (p. 15).

Principals from 128 Massachusetts schools responded to a survey that requested the number of distinct tracks their respective schools had offered in ELA, history, science, and math over the previous five years. To supplement the survey data, Loveless (2009) also collected achievement data from 1995, 2005, and 2008. Table 1 (Loveless, 2009, p. 17) displays the data from 1995 and 2009. The vast majority of schools had detracked in ELA, history, and science by 2009. Math, however, continued to be commonly tracked although the number of tracks had decreased.

Table 1

Percentage of Middle Schools with Tracking in Academic Subjects

	1995			2009		
	1 level	2 levels	3+ levels	1 level	2 levels	3+ levels
Math	15.2%	30.3%	54.5%	15.6%	49.2%	35.2%
ELA	55.1%	14.9%	30.0%	72.7%	22.7%	4.7%
History	67.7%	9.4%	22.9%	89.8%	7.0%	3.1%
Science	62.2%	14.2%	23.6%	86.7%	9.4%	3.9%

Several factors were statistically significant in influencing whether or not a school had detracked. First, schools with higher levels of free or reduced lunch were more likely to detrack; suburban schools were less likely to detrack than urban schools. Second, schools with grades 7-8 maintained tracking at higher levels than schools with grades 5-8 or 6-8. Last, schools with more parental involvement and influence in school decisions tended to have higher levels of tracking.

To answer the last research question, Loveless (2009) compared scores on the Massachusetts Comprehensive Assessment System (MCAS) test. While ELA scores show no difference between tracked and detracked schools, math scores tell a different story. Schools with tracking had significantly higher rates of students in the “advanced” and “proficient” categories and lower rates of “needs improvement” and “failing” compared to detracked schools, as shown in Table 2 (Loveless, 2009, p. 23).

Table 2

Distribution of Achievement in 8th Grade Math

Number of math tracks	% Advanced	% Proficient	% Needs Improvement	% Failing
1 (n=17)	15.8	29.3	28.8	26.2
2 (n = 66)	18.6	31.9	28.7	20.7
3 (n=43)	26.6**	34.5*	24.1**	14.8**

Note. *p<.05 **p<.01

Loveless (2009) controlled for school-level socioeconomic status (SES) and found that there was a three percentage point improvement for each additional track offered (up to three) in eighth grade. “That means a school with 200 eighth graders that offers at least three levels of math is typically attended by twelve more students scoring at the advanced level than a detracked school of similar size and SES status” (p. 6).

While it used to be common to differentiate course plans based on a student's ability level, motivation, and career interests, this practice proved to increase harmful socio-economic gaps within our society. Students from wealthier households tended to take more advanced coursework that prepared them for college, while students from low-income households, especially students of color, tended to take lower-level classes. To curb the growing divide while also attempting to boost the United States' ranking in international comparisons, the educational policy practices have shifted. Now, the curriculum path requires nearly all students to be exposed to advanced levels of mathematics. New York and Texas, in 2001 and 2003, respectively, led the way by requiring all students to complete college preparatory math courses. At a local level, Chicago Public Schools (CPS) enacted a policy in 1997 that eliminated remedial courses and required all students to take a college preparatory course sequence.

Allensworth et al. (2009) studied the effects of Chicago Public Schools' policy, particularly focusing on those students who would have otherwise enrolled in remedial math classes. The authors chose CPS because it allowed them to easily eliminate a large amount of selection bias, since this policy applied to all students in a very large metropolitan area who attended numerous different schools. It also allowed the authors to study within the context of a low-performing setting, whereas many other studies were performed at such a broad level that it was difficult to generalize the results to this particular setting. The authors posed three research questions: 1) "To what extent did enrollment in ninth-grade college preparatory courses increase as a result of the policy mandate, and how did the social distribution of course taking by students' race, ability level, and disability status change between pre- and post policy periods?" (p. 373); 2) "Did the academic outcomes of students improve by taking college prep classes instead of remedial classes, and did the effects differ by their academic abilities as they began high school?" (p. 373); and 3) "To what extent did the policy affect students' academic outcomes overall, and how did the effects differ for students entering high school with different abilities?" (p. 373).

This study specifically looked at the policy that required all students to take Algebra 1 and English 1 in ninth grade. For this review, only the results and conclusions relevant to Algebra 1 are discussed (unless unavoidable). During this study, there were 59 high schools in the CPS system. Data was collected over a ten-year period from 1994 to 2004, and the ninth-grade cohorts ranged in size from 21,587 to 26,197. Allensworth et al. (2009) divided participants into groups based on expected course enrollment (had the policy not been in effect). For example, some students and even some schools were already planning to enroll in Algebra 1 and thus their curriculum plans were largely unaffected by the change. These groups also allowed for control over potential influences from other policies enacted around the same time that may have an unwanted impact on the data.

Upon analyzing the data for research Question 1, Allensworth et al. (2009) concluded that the policy affected curriculum plans for low-level students the most, while it had virtually no effect on high-ability students. Racial and socioeconomic course-taking gaps that were present pre-policy were no longer present post-policy. The authors cautioned, though, that it is reasonable to assume that some schools simply renamed their remedial classes to fit the policy without actually integrating a heterogeneous learning environment. With that possibility being out of their control, though, the authors proceeded with the study.

When looking at the performance of students post-policy, Question 2, there was a 10% increase in Algebra 1 credits earned for every 20% increase in enrollment (indicating a 50% pass rate among low-level students). However, failure rates increased among both low-level and average-level students, with rates dropping three percentage points and 8.9 percentage points, respectively. Average-ability students also took the hardest hit in decreasing grades, with an average drop of 0.18 grade points. Furthermore, average-ability students were absent an average of 1.6 days more than pre-policy. Test scores were unaffected, but the authors noted that only 7 of 48 questions on the standardized test used by CPS tested Algebra knowledge.

To answer Question 3 as well as provide more context and clarity to research Question 2, Allensworth et al. (2009) compared schools that did and did not offer remedial courses pre-policy. The results were similar but slightly different. They showed that low-level students were the most affected by the policy, which makes sense since their curriculum track was the most impacted. Compared to pre-policy years, the two groups of low-level students were 8.8 and 7.4 percent more likely to earn Algebra 1 credit in ninth grade, but they also had a 7.4 percent increase in failure rates. This seems to indicate a bit of a polarizing effect on the low-level students. Under this model, the number of absences from the average-ability group increased from 1.6 days to 3.14 days. Under this model, low-level students saw a math grade decline of 0.15 points.

Lastly, Allensworth et al. (2009) extended their work to examine the long-term outcomes for students pre- and post-policy. No finding was consistent across all ability groups, and much of the data could have possibly just been “noise.” Regardless, the authors noted a few statistically significant findings. Students placed in the lowest-ability group had a slightly higher

GPA over their high school careers and were slightly more likely to graduate. Students placed into the low-ability group experienced a 2.8 percent decrease in 4-year college enrollment. The authors concluded that the policy change eliminating remedial math courses in favor of Algebra 1 did not prepare more students for college, neither by way of higher-level math completion nor four-year college attendance, at least in terms of large urban settings with chronic low performance.

An increasing number of studies have been conducted regarding the benefits experienced by taking more rigorous coursework. However, Nomi (2012) pointed out that many of these studies compare students who took different courses, such as remedial math vs. Algebra 1. Few studies, the author argued, have considered the effects of universalizing rigorous math courses for all despite the wide range of ability levels. With a turn away from tracking or grouping students by ability, there is still much to be learned about the effects of organizational decisions that result from an algebra-for-all curriculum. The author argued that two main groups of students in particular needed to be studied: 1) students who would otherwise be enrolled in remedial courses and thus potentially be pushed beyond their readiness level, and 2) high-level students who would normally take Algebra 1 anyway. How this high-level group would be affected by the addition of lower-level peers is of particular interest. With that focus, Nomi (2012) posed the following research questions: 1) “To what extent did the algebra-for-all policy in Chicago affect classroom academic composition?” (p. 490); 2) “For high-skill students who were not targeted by the policy, how did the policy affect math achievement through changes in academic composition in algebra classrooms?” (p. 491); “What was the effect of academic composition on math achievement among high-skill students?” (p. 491).

In 1997, Chicago Public Schools (CPS) implemented stricter graduation requirements, thus requiring all students to take Algebra 1 in ninth grade. Students who would have been enrolled in remedial math in ninth grade were now forced to take Algebra 1 instead. While this provided all students with the opportunity to take a more advanced course, the policy was found to have not improved test scores in those who would have otherwise taken remedial math. In addition, failure rates among students with average skills increased after the 1997 policy implementation. This study, performed by Allensworth et al. (2009), provided an insight to how the lower-level students were affected, the effects on the high-skilled students were not studied.

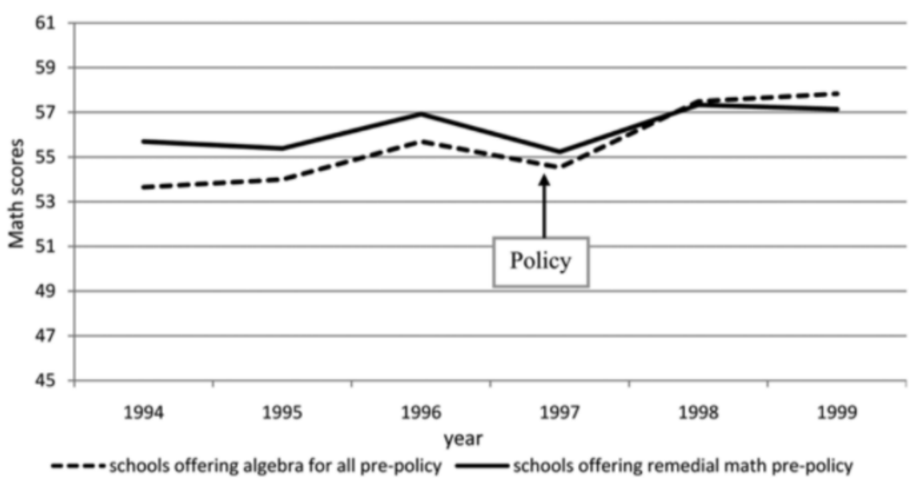
For the study, Nomi (2012) used schools who had offered Algebra 1 for nearly every student, even before the policy implementation, as the control group. The author aimed to be able to use this group to factor out the impacts of other CPS policies, as these policies would theoretically impact schools in both groups the same way. The study examined 18,005 ninth-grade students from 58 schools who were separated into six cohorts; half of the cohorts were assigned to the control group. Nomi (2012) defined high-skill students as “those whose incoming math abilities are more than .7 standard deviation above the overall average” (p. 493). This means that those students scored at or above the 60th percentile in math on the Iowa Tests of Basic Skills (ITBS). Scores from the Tests of Academic Proficiency, administered in the spring of students’ ninth-grade year, were then used to measure high school achievement.

When data from 1994 through 1999 was analyzed, there was a stark difference in academic classroom composition between the test cohorts and the control cohorts. Peer ability in schools who had already offered Algebra 1 pre-policy showed a slight change, while schools which had traditionally tracked lower-level students into remedial math classes saw a large decline for high-skill students and an improvement for low-level students. This wasn't surprising, as the detracking caused a mixed-ability classroom.

The results for research Question 2, test scores were 2.04 points higher for high-skill students in tracking schools than those in the control group. In fact, test scores for both types of schools generally increased during the pre-policy years. In 1998 and 1999, however, scores changed dramatically. High-skill students from the control group saw test score improvements of 3.84 and 4.18, respectively. High-skill students from the test cohorts, however, only saw gains of 1.64 in 1998 and 1.44 in 1999. Figure 1 (Nomi, 2012, p. 499) below shows these trends.

Figure 1

Trends in the Average Ninth-Grade Math Test Scores for High-Skill Students



Nomi's (2012) final analysis measured the effect of classroom academic composition on math outcomes. An instrumental variable analysis revealed that as peer skill levels decreased for high-skill students in the test group, their test scores were expected to also decrease. For example, if high-skill students' peer ability levels decreased by 1 standard deviation, their test scores were expected to decrease by about a quarter of a standard deviation, or 3.15 points.

In conclusion, Nomi (2012) showed that while tracking has been widely criticized when looking at low-level students, universalizing the curriculum seemed to hold high-skill students back from reaching their full potential. The author hypothesized that this could have been due in part to the need to "water down" the curriculum in order to reach the low-level students. Nomi (2012) noted that simply mandating a policy pushing advanced curriculum is not sufficient for low-level, nor high-level students, and additional supports need to be put in place.

Sorensen et al. (2018) looked at the consequences of students who took Algebra 1 during their eighth-grade year. Citing limited data sets and short-term research questions in previous studies, the authors asked the following questions:

> 1. What is the "impact" of placement into eighth-grade algebra on short-term outcomes (attendance, math course performance) across the transition to high school, and on longer-term high school milestones (completing math course requirements, graduation)?
>
> 2. To what extent do "impacts" of placement into eighth-grade algebra vary for students with different levels of prior readiness? (p. 2)

The authors drew from two large, diverse school districts for their study: California's Elk Grove Unified School District (EGUSD) and Illinois' Chicago Public Schools (CPS). Data from six EGUSD cohorts who graduated between 2011-2016 and four CPS cohorts who graduated between 2013-2016 together provided 123,057 data points. Since approximately one-third of

CPS schools do not offer Algebra 1 in eighth grade, the authors used a propensity weighting design to analyze the data. This reduced the possibility and potential impact of selection bias.

Sorensen et al. (2018) found at both sites students who had low achievement were rarely placed into an Algebra 1 setting. Students who took Algebra 1 in eighth grade saw a decrease in GPA compared to peers who did not take Algebra 1. The authors attributed this to the increased rigor and difficulty level. For those students, though, GPAs generally rebounded by the end of ninth grade and in Chicago tended to be higher than their non-Algebra peers. Furthermore, students who took eighth-grade Algebra 1 had better attendance than their peers, though this data was statistically insignificant for CPS students.

Regarding long-term outcomes, Algebra 1 students were more likely to complete Algebra 2 by eleventh grade which allowed them to continue on to more advanced math courses. While graduation rates were unaffected in EGUSD, CPS students saw higher graduation rates, particularly for students with average or above-average grades and test scores going into eighth grade. Ultimately, Sorensen et al. (2018) determined there were no negative impacts on average and above-average students taking Algebra 1 in eighth grade, though there was not enough data to make a conclusion on low-performing students due to lack of enrollment in the course.

While a correlation has been well documented that students who take Algebra 1 in eighth grade have better outcomes later in life than those who do not take Algebra 1 in eighth grade, research has been limited regarding the causal impact of earlier algebra. Clotfelter et al. (2015) analyzed such initiatives in two large North Carolina school districts during the 2002-2003 school year, hoping to avoid selection bias that was common in other studies. One district reversed their policy after two years, ceasing the push for early Algebra 1, while the other continued with the policy. Additional districts that did not enact a policy change were incorporated to provide extra statistical assurance as “control districts.” Clotfelter et al. (2015) studied the effects of eighth-grade Algebra 1 by looking at standardized test scores in Algebra 1, passing rates on Geometry and Algebra 2 standardized tests, the likelihood of a student progressing to Calculus in high school, and the likelihood of a student repeating Algebra 1.

Data was collected from North Carolina Education Research Data Center longitudinal records and focused on students who had been enrolled in seventh grade between the 1999-2000 and 2004-2005 school years. Participants attended a school in Charlotte-Mecklenburg, Guilford, or one of the control districts and yielded a sample size of 135,752 students.

After testing for reliability, the authors found that students in accelerated programs scored 45% of a standard deviation lower on the Algebra 1 end of course (EOC) standardized test than students in the control group. Accelerated students also were 11% less likely to pass the Geometry EOC exam by 11th grade. Acceleration also seemed to have a larger negative impact on students eligible for free or reduced lunch. Algebra 2 performance did not appear to have been impacted by accelerated Algebra 1. Clotfelter et al. (2015) then tested the possibility of negative impacts being the result of “transition cost,” or the temporary decline immediately following a large change, to determine whether that had been the reason for the negative impacts. If there had been a transition cost, the authors hypothesized that Charlotte-Mecklenburg (the district that reversed policy) would have seen more negative effects than Guilford. However, the data failed to support that hypothesis which disproved transition cost as a factor.

Next, Clotfelter et al. (2015) used transcript data to determine whether accelerated Algebra 1 resulted in a higher rate of enrollment in Calculus. They found that advanced students, as defined by 6th and 7th grade test scores, who took Algebra 1 in 8th grade went on to take Calculus 80% of the time, as compared to 60% if Algebra 1 had been taken in 9th grade. This difference was due to the student having an “empty year” in the curriculum track and not needing to “double up” on math courses. Students performing in the middle of their cohort only saw a small increase in Calculus enrollment, from the single digits to the teens. The study found that over 80% of these students fell off track at some point, thus not giving them time to take Calculus, and most students in this group did not “double up.” Students in the lowest-performing group had essentially no chance of enrolling in Calculus regardless of when they took Algebra 1.

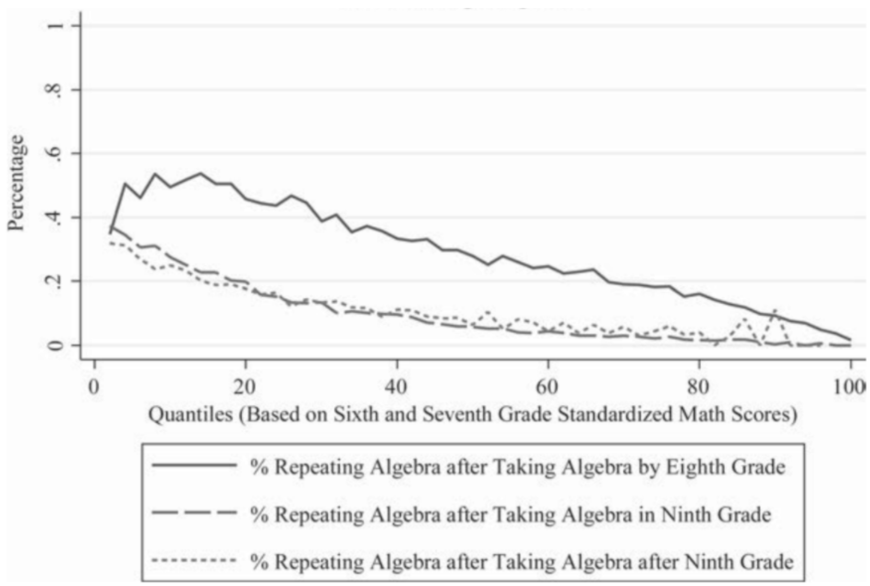
Lastly, Clotfelter et al. (2015) looked at retake rates. Figure 2 shows that while rates of failing were virtually the same when comparing students who had taken Algebra 1 in 9th grade with those who had taken it after 9th grade, rates of accelerated Algebra 1 were significantly higher across all ability levels (p. 184).

For students in the 20th percentile, retake rates were 50% for those in accelerated settings as compared to 20% for those who took it in high school. This observation, Clotfelter et al. (2015) noted, “indicates that enrolling the lowest-performing students in early Algebra 1 introduces significant downside risks with little to no upside potential. For moderately performing students, the potential rewards are moderate and the downside risk is significant” (pp. 183-184). The only group that seemed to benefit more than suffer was the advanced group, as this gave them the opportunity to take Calculus without the need to “double up” courses.

Therefore, the authors concluded that accelerating Algebra 1, without broader reform across the whole curriculum, significantly hampered performance in both Algebra 1 and Geometry and actually increased inequality across ability levels.

Figure 2

Probability of Repeating Algebra 1 Conditional on First Taking Algebra 1 in a Specific Grade, by Percentile Rank of Mean Sixth/Seventh Grade Math Score, Ten District Sample



Crawley (2018) studied the potential impacts of Fort Lee Public Schools' decision to discontinue tracking for Algebra 1 beginning with the 2015-2016 school year. Prior to this policy change, students were put on course plans, or "tracks," based on ability and test scores. Fort Lee's new policy eliminated eighth-grade pre-Algebra and instead required all eighth-grade students to take Algebra 1 except for the top five percent who were advanced to Geometry. The goal for this policy shift was to allow more students the opportunity to take advanced math courses, while hopefully not sacrificing students' long term success. Traditionally, Algebra 1 has been a course taken in the ninth grade. Recently, though, states have shifted toward eighth-grade Algebra 1 policies, of which California and Minnesota were the leaders in 2011. Rationale for this shift commonly focuses around studies that have shown advanced math participation results in higher rates of college enrollment and higher career earnings.

This study followed two cohorts of students at Fort Lee Public Schools, a suburban district of around 4,000 students located in New Jersey. Nineteen percent of students were considered economically disadvantaged (determined based on the free and reduced lunch program). At the time of the study, the school was known as a high performing school, and ranked 55th in the state and regularly sent students to Ivy League schools. The two cohorts in this study attended the same middle school; the only difference was the fact that they were one year apart. The first cohort took a pre-Algebra course in eighth grade, followed by Algebra 1 in ninth grade. One hundred twenty-four students in this cohort qualified to be used in research question 1 (R1) and 115 qualified to be used in research question 2 (R2). The second cohort followed the new policy where Algebra 1 was moved to eighth grade. One hundred twenty-one students from this cohort were used in R1 and 110 were used in R2.

In order to properly study the effects of required Algebra in eighth grade, Crawley (2018) noted that students' race, socio-economic status, gender, attendance, and prior math achievement also needed to be included in the data, since all have been proven to have an impact on one's math achievement, either directly or indirectly. For each research question, the author analyzed scores from the Algebra 1 and Geometry Partnership for Assessment of Readiness for College and Careers (PARCC) Assessments taken in 2016 and 2017, respectively, and conducted an ANOVA on each variable to determine its significance.

Crawley (2018) proposed two main research questions, with four symmetrical sub-questions each. These questions analyzed performance at the end of that year, as well as the following year after the students had completed Geometry. Question one (R1) was "How does the shift from offering Algebra as an advanced eighth-grade course to mandating it universally as the standard eighth-grade math course affect student achievement, measured by performance on the Algebra 1 PARCC end-of-course assessment?" (p. 25). Question two (R2) was, "How does the shift from offering Algebra as an advanced eighth-grade course to mandating it universally as the standard eighth-grade math course affect student achievement, measured by performance on the Geometry PARCC end-of-course assessment?" (p. 25).
The symmetrical sub-questions asked the main question with focus on the achievement of (a) economically disadvantaged students, (b) black and Hispanic students, (c) female students, and (d) male students. This resulted in a total of ten research questions. Attendance and prior math achievement were also considered throughout all research questions.

Crawley (2018) found that in regard to R1, the adoption of the policy had a negative impact on student performance, as demonstrated by the results of the Algebra 1 PARCC exam. However, the author was sure to note that the goal of the policy was not to improve students' Algebra 1 assessment scores. In fact, a slight decrease was expected due to students losing a year of foundational math. Rather, the goal of the policy was to advance students through Algebra 1, Geometry, and Algebra 2 coursework sooner, so that they have more opportunities to take higher level math courses. This potential immediate negative impact was deemed to be an acceptable tradeoff for future opportunities.

All sub-questions of R1 showed eighth-grade Algebra having no statistically significant effect on students' PARCC exam scores, with the exception of black and Hispanic students, who had an unexpected positive change in performance. Furthermore, black and Hispanic students who were also considered economically disadvantaged also saw a positive effect of the policy. For all other sub-questions, attendance and prior math achievement proved to have the most influence over PARCC scores, not the policy change.

R2 revealed no statistically significant impact on students' Geometry performance, and this finding extended throughout all four sub-questions. There were also few differences with the results of the sub-questions when compared to the Algebra 1 results. While attendance and prior math achievement had an influence on Algebra 1 success among economically disadvantaged students, there was no statistically significant influence in Geometry. Also, the benefit that was shown in the black and Hispanic subgroup disappeared in the Geometry PARCC data. Furthermore, while prior math achievement remained a strong influence on Geometry success, attendance was only significant when looking at the whole cohort and the male subgroup.

Based on the data, the author concluded that while moving Algebra 1 to eighth grade may have had a slightly negative effect initially, the long term negative effects seemed to disappear relatively quickly. In fact, Crawley (2018) theorized the small initial drop in performance could be mitigated by efforts to increase attendance and prior achievement, as these two factors seemed to have a greater influence than any other factor.

The author did acknowledge some limitations and areas for further research. The study population, while very easy to compare between cohorts, was limited in number. Therefore, larger studies ought to be conducted to confirm the findings of this study. Another limitation was the short-term nature of the study. Crawley (2018) encouraged other studies to evaluate data throughout the students' high school career to determine if eliminating a year of foundation math had negative impacts further down the line.

In 1999, the California Senate passed the Public School Accountability Act (PSAA), which penalized schools for allowing students to take general math courses in eighth grade, including pre-Algebra. This act forced eighth-grade Algebra 1 enrollment to jump from 16% to 51% from 1999 to 2008. In 2008, the California Board of Education passed a policy that essentially made Algebra 1 required in eighth grade if the school wished to meet expectations of the No Child Left Behind Act. After pushback in the court system and the state's adoption of the Common Core State Standards, the need to require Algebra 1 in eighth grade recessed. However, many schools anticipated the 2008 policy would become stricter and funneled the majority of their eighth-grade students into Algebra 1. Due to these policy inconsistencies, there was a wide variety of Algebra 1 enrollment rates.

Domina et al. (2014) used district-level data from all K-12 public schools in California starting with the 2003-2004 school year and ending with the 2009-2010 school year to study the consequences of the 8th-grade Algebra 1 push on 10th-grade performance as shown on the California High School Exit Exam (CAHSEE). The CAHSEE is a statewide test taken in the spring of a student's tenth-grade year; passing it is a requirement for graduation. During the studied years, 222 school districts averaged around 300,000 eighth-grade students per year. In the first year of the data, approximately 40% of students took Algebra 1 or higher. This number increased to 60% by 2009-2010, which was higher than the 35% enrollment rate reported by the National Assessment of Educational Progress (NAEP) in 2010.

After controlling for several variables and dividing school districts into groups based on size, Domina et al. (2014) discovered that accelerating Algebra 1 to eighth grade had minimal effects on student achievement. However, in school districts with 850 or more eighth-grade students enrolled annually, Algebra 1 acceleration resulted in a sizable negative impact on student achievement. For every 1 standard deviation increase in Algebra 1 enrollment, CAHSEE scores fell by 0.05 to 0.07 standard deviations. Since more than 80% of students analyzed attended schools in these large districts, these results were cause for concern. Domina et al. (2014) suspected that the reason for large districts' struggles was due to logistical challenges, such as the greater need to shuffle teaching assignments and course organization.

The rate of California students taking Algebra 1 in the eighth grade rose from 32% in 2003 to 59% in 2011. As a result, there was a need to study the impacts of this movement on future student achievement.

Laing et al. (2012) used scores from the California Standards Tests (CSTs) to investigate the following research questions:

1) During a 4-year period from grades 8 through 11, how many students took the CST for Algebra 1 in 8th grade? In subsequent years, how many of these students continued taking higher level math CSTs? How did students perform on each of the CSTs for each of the subsequent years?

2) For those eighth graders who took the CST for Algebra 1 and scored below proficient, what CSTs for mathematics did these students take in ninth grade and how well did they perform?
3) For those eighth graders who took and scored proficient or above on the CST for General Mathematics, what CSTs for mathematics did these students take in ninth grade and how well did they perform?
4) Can student performance on the CST for Grade Seven Mathematics predict which CSTs he or she will take at eighth grade between General Mathematics and Algebra 1? (p. 330)

Liang et al. (2012) identified 456,392 students who were seventh graders in 2006 and became eighth graders in 2007 as well as 471,481 students who were eighth graders in 2006 and became ninth graders in 2007. For research questions 2 and 3, the authors focused on the second group of students. They divided that cohort into four subgroups: (1) those who were proficient or above on the Algebra 1 CST, (2) those who were below proficient on the Algebra 1 CST, (3) those who scored proficient or above on the General Mathematics CST, and (4) those who scored below proficient on the General Mathematics CST. The authors assumed students in subgroup 1 had been correctly placed and students in subgroup 4 had little chance of success in Algebra 1. Therefore, their main focus turned to students in subgroups 2 and 3. Since 64% of students in subgroup 2 took the Algebra 1 CST again in ninth grade and 83% of subgroup 3 took the same CST in ninth grade, albeit for the first time, the authors were able to easily compare success rates on the CST.

From 2003 to 2008, the number of eighth-grade students taking the Algebra 1 CST increased by 19%; however, rates of higher level CSTs did not increase at the same rate. Ninth graders taking Geometry only increased about 8%, tenth-grade students taking the Algebra 2 CST increased about 6%, and eleventh-grade students taking the Summative High School Math CST increased 7%. The disparity in rate increases between eighth-grade Algebra 1 and the other courses raised questions about the effectiveness of the policy shift. The authors then turned to the last part of research question 1 to provide clarity. Data on passing rates showed increases in passing rates of 3%, 4%, 4%, and 7% for the Algebra 1, Geometry, Algebra 2, and Summative High School Mathematics CSTs, respectively. Since these numbers were consistent, they found no answer for the aforementioned rate gap, and the authors continued to research questions 2 and 3 for clarification.

Of the 201,698 eighth-grade students who took the Algebra 1 CST in 2006, 58.19% scored below proficient (subgroup 2). Eight percent of this subgroup went on to take the General Mathematics CST the following year, and for every student who scored proficient or above, seven students scored below proficient. The majority of subgroup 2, 64%, took the Algebra 1 CST again as ninth graders. For every proficient student, there were 5 who once again scored below proficient. Of the 26% of subgroup 2 who went on to take the Geometry CST the following year, for every proficient student there were 10 students who were below proficient.

Regarding research question 3, of the 188,482 eighth-grade students who took the General Mathematics CST in 2006, 27.51% were proficient or above (subgroup 3). Around 12.5% of students in subgroup 3 took the General Mathematics CST again in ninth grade; proficiency occurred at a nearly 2 to 1 ratio as compared to those who scored below proficient. Laing et al.

(2012) could not definitively answer why there had been a drop in success for those who retook a test on which they had performed well on the year prior. The two main reasons given by the authors were lack of access to transcript data and the test’s reliability estimate of .88 which indicated a chance of fluctuation from year to year. Nevertheless, of the 83% of students in subgroup 3 who took the Algebra 1 CST in ninth grade, the ratio of proficient students to non-proficient students was 3 to 5. This rate was significantly higher than the overall performance across all ninth-grade students. Also, 4% of subgroup 3 students took the Geometry CST and had a proficient to non-proficient ratio of 1 to 2, once again besting the overall success rate. These results indicated that subgroup 3 students were generally more successful than students in subgroup 2; and “bubble” students may be better off not taking Algebra 1 in eighth grade.

Finally, the analysis of research question 4 found that the greatest predictor of whether a student would take the General Mathematics or Algebra 1 CST in eighth grade was their seventh-grade mathematics CST score. This was not surprising to the authors. What followed, however, was noteworthy. Parent education, historically correlated with socioeconomic status, did not play a significant role in determining student placement. This, the authors noted, is contrary to what many policymakers argue is the problem with large-scale standardized tests. Instead of socioeconomic status, the second major predictor of eighth-grade placement was the students’ English/Language Arts (ELA) CST scores in seventh grade. Therefore, Liang et al. (2012) concluded that one could predict eighth-grade placement based on seventh-grade CST scores in Mathematics and ELA.

Gamoran & Hannigan (2000) conducted a study to add evidence to the "Algebra for Everyone" pool of data. Prior to this movement, students in ninth grade had been typically sorted into tracks based on their math ability with options that included General Math, pre-Algebra, Algebra 1, and Geometry. While students in the Algebra 1 or Geometry tracks tended to succeed and achieve success in college, students with lower math abilities struggled to complete Algebra 1 and Geometry. In fact, only 63% of students who had entered high school in 1988 had completed those two courses four years later. In response, Louisiana led the charge to make Algebra 1 mandatory for graduation, and many states followed suit. Despite large policy reform, the authors noted that many assumptions have been made regarding its efficacy. Therefore, Gamoran & Hannigan (2000) posed the following research question: "Do students benefit differently from taking high school algebra? Or do all students benefit similarly, regardless of their prior math performance?" (p. 243).

This study used the first two cohorts of the National Educational Longitudinal Study (NELS). The NELS began with students enrolled in eighth grade in 1988 and reassessed them in 1992. This study provided a sample of 12,506 students from approximately 1,000 schools across the nation. The authors also sorted the students into four groups based on transcript data: students who took no Algebra 1, students who took Algebra 1 in eighth grade, students who took Algebra 1 in ninth or tenth grade, and students who took Algebra 1 in eighth grade and high school. The main focus was to compare students who took no algebra with students who took Algebra 1 in ninth or tenth grade (though all students were studied). To reduce bias from unmeasured differences, the authors considered eighth-grade science, reading, and history scores as well. Gamoran & Hannigan (2000) also controlled for differences such as sex, race, English proficiency level, and socioeconomic status (SES).

First, the authors analyzed the sociodemographics of the four groups. Those in the “no algebra” group had the lowest SES and had an above average population of males, African Americans, and Latinos. Females, Whites, and Asian Americans had below-average participation in this group. Conversely, students who took Algebra 1 in eighth grade tended to be from high SES households and were largely females, Whites, and Asian Americans. While these results followed the traditional thinking, it did not shine light on the research question.

After a series of regression models, Gamoran & Hannigan (2000) recorded four main findings. First, students who took Algebra 1 in high school gained 3.64 points more on the math test than students who took no algebra. On average, this was approximately half of the growth that takes place between grades 8 and 10. Second, students who took Algebra 1 in high school, regardless of prior math knowledge, benefited similarly; there was no indication that students with high math skills benefited more than those with low skills. Third, when the authors included a variable for eighth-grade students in the 20th percentile or below, taking Algebra 1 in high school benefited those students less than their peers. Despite less impact, these students still benefited more than peers who took no algebra. Fourth, students who took Algebra 1 in eighth grade, yet still entered high school with low test scores, did not see a benefit on their tenth-grade test scores. These results led Gamoran & Hannigan (2000) to conclude that “all students, regardless of prior math skills, benefit from taking high school algebra” (p. 250).

Non-Traditional Math Initiatives

While there is an increasing push for students to take more advanced math courses, it is also widely acknowledged that many students are not prepared for the associated rigor. As measured by the 2005 NAEP, 30% of eighth-grade students were below basic level, and only 30% of students were at or above a proficient level. These rates were even worse in Chicago Public Schools (CPS), where half of the ninth-grade students enrolled failed at least one course. CPS's 1997 policy decision to eliminate curriculum tracking and require all students to take Algebra 1 in ninth grade was intended to help the low-level students increase their mastery. However, this policy proved to be challenging for teachers due to the new heterogeneous mix of low- and high-level learners in the same classrooms. The benefits were questionable, and some data even revealed a decline among high-level students because of the change in classroom environment. School systems elsewhere have attempted to find a middle ground by improving low-level instruction while keeping some form of academic homogeneity. One example in Rochester, New York, replaced remedial math courses with a course that taught the same college-preparatory material, just at a slower pace. However, any evidence of improved academic outcomes was largely inconclusive.

In 2003, CPS built on their 1997 college-preparatory policy by implementing "double-dose" algebra. This policy enrolled ninth-grade students who scored below the national median on their eighth-grade standardized math test in two periods of Algebra. One of the periods was the college-preparatory Algebra 1 course required under the 1997 policy, while the second period was an algebra support class. The district provided two curricular options as well as professional development for teachers. CPS also set guidelines that (1) students' algebra support teacher should also be their Algebra 1 teacher, (2) the courses should be offered sequentially, and (3) classmates in the algebra support class should be enrolled in the same period of Algebra 1. In the first year, condition one was met 80% of the time, condition two was met 72% of the time, and condition three was met 92% of the time. All three conditions experienced a decline in adherence in future years as the policy was adjusted. An unintended consequence of this policy was that it forced a shift back to a more homogeneous classroom in order to streamline scheduling.

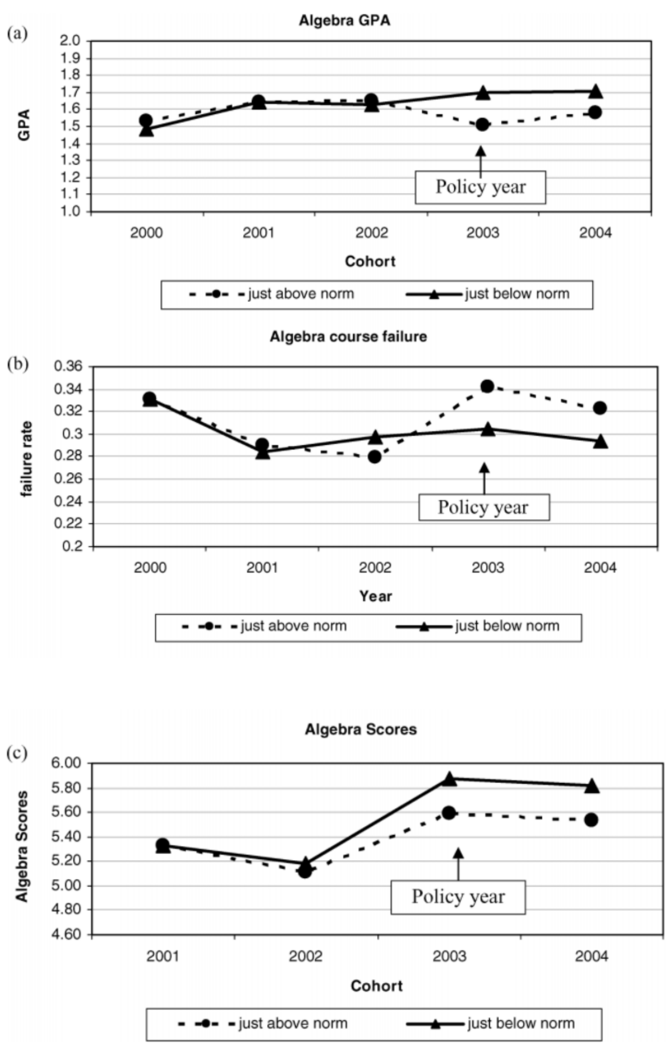
Nomi & Allensworth (2009) posed the following research questions: 1) “Did putting lower performing students in double-dose algebra lead to significant improvement in their math achievement, measured by test scores, grades, and pass rates?” (p. 119); 2) “Did the policy affect the academic outcomes of students who were not targeted by the policy (i.e., those entering high school with math scores above the national median)?” (p. 119); and 3) “How did policy effects differ by students' incoming math skills?” (p. 119).

The authors took data from ninth-grade students enrolled in CPS between 2000 and 2004. This resulted in data from 92,432 students across 64 high schools; which drew its information from U.S. census data, school records, transcripts, Iowa Tests of Basic Skills (ITBS) scores from third to eighth grade, and the PLAN exam taken in the fall of tenth grade.

Nomi & Allensworth (2009) found that the double-dose strategy did not produce the intended results. While students saw a net test score increase of 0.56 points with students just below-average performing the best at 0.76 points, these improvements seemed to disappear when the students took Geometry. Furthermore, there was a net grade decline of 0.05 GPA points and a three percent increase in failure rates across all cohorts. High-level students and very-low ability students were hit particularly hard; both experienced above-average grade declines as well as four percent failure increases. Figure 3 below shows the performance of students just-below and just-above norm both pre- and post-policy.

Figure 3

Trends After Double-Dose Policy Implementation



Note. (a) trends in student GPA before and after policy, (b) trends in Algebra 1 failure rates before and after policy, and (c) trends in test scores before and after policy (Nomi & Allensworth, 2009, p. 135).

An unintended consequence of this policy was that it reverted schools back into a form of tracking. As high-level students had been placed together more often, the authors hypothesized that expectations and perceptions by both students and teachers may have changed, which may explain why high-level students' grades tended to decline post-policy. In conclusion, Nomi & Allensworth (2009) stated, "Failure rates remained the same for targeted students, and the policy unintentionally increased failure rates in ninth-grade algebra for students who were not targeted by the policy. Many people in the district viewed the policy as a failure, and conversations with district personnel suggest that the policy was widely criticized" (p. 142). As a topic of future research, the authors suggested exploring the mechanisms of the double-dose policy that affected outcomes such as the motivation changes displayed from a student who had an elective course replaced with another period of math.

As an extension of their 2009 study, Nomi & Allensworth (2013) explored the "why," or mechanisms, behind their 2009 results. Three mechanisms were identified that could have affected the students influenced by the double-dose algebra policy: "expanded instructional time; improvements in instruction resulting from curricular resources, professional development, and expanded instructional time for teachers; and ability grouping into more homogeneous classes" (p. 762). In theory, low-level students would be helped by this policy because of additional time and support provided to master the curriculum. Also, the policy should have helped high-level students as the classroom would have now offered fewer distractions, fewer discipline problems, and more challenging instruction. The authors noted, however, that for high-level learners this shift may have had a detrimental effect due to the "fish pond effect," a phenomenon in which teachers assign higher grades to students that appear better than their classmates. With low-level learners removed from that classroom, the perception of "average" would have logically and drastically shifted.

In consideration of the identified mechanisms as well as the "fish pond effect," Nomi & Allensworth (2013) asked three research questions.

> RQ1) To what extent did the sorting that resulted from the policy affect algebra test scores and pass rates among students with below-norm and above-norm skills? To what extent were failure rates affected by the overall skill level in the classroom versus students' own abilities relative to their classroom peers (i.e., "fish pond effects")?
>
> RQ2a) For above-norm students post-policy, how did the outcomes differ between students who took algebra in mixed-skill classrooms with below-norm students receiving support and those in homogeneous classes without such below-norm students?
>
> RQ2b) Similarly, for below-norm students, how did the outcomes differ between those who took double-dose algebra in mixed-skill classrooms and homogenous classes with all below-norm students?
>
> RQ3a) In what ways did the policy, and the ability sorting induced by the policy, affect students' classroom climate and instructional experiences for below-norm and above-norm students?
>
> RQ3b) How were these changes in the classroom environment related to students' test scores and pass rates? (pp. 763-764)

For this analysis, the authors used two cohorts of students. The pre-policy cohort was the ninth-grade students in the 2002-2003 school year, and the post-policy cohort was the ninth-grade students in the 2004-2005 school year. With some small exceptions, all general education students (n= 24,259 in 55 schools) in those cohorts were used in this study (students receiving special education services were exempt from the double-dose policy). The population shrunk to 6,779 when analyzing survey results for research question 3. This discrepancy in population numbers led the authors to conduct tests to determine whether response bias existed. None was found, and respondents were generalized to the general population.

Nomi & Allensworth (2013) found that test scores largely rose across the board. Students with above-norm skills saw a test score increase of 1.07 while below-average students' scores improved more than expected when compared to pre-policy students. The authors also discovered that higher-level students benefited from higher-skill peers much more than low-level students did. Therefore, the drop in class skill level had little negative impact on below-norm students' test scores.

Pass rates, however, painted a different picture than test scores. While pass rates slightly improved for below-norm students, students just above the threshold for placement in single-period Algebra 1 saw pass rates decrease from 69% to 63%. This provided support for the "fish pond" theory; that is, students who may have been toward the top of their class pre-policy were now toward the bottom post-policy. Below-norm students saw a slight increase in pass rates, from 69% pre-policy to 71% post-policy.

In regard to the classroom environment and instruction, both above- and below-norm felt there was an increase in academic demand. For above-norm students, this was likely due to teachers assuming their students can handle more rigorous material. Another reason was that students just above the cutoff perceived their classes to be more demanding due to the relative skill level of their peers. For below-norm students, increased demand was likely due to teacher implementation of provided instructional resources and professional development strategies. With more time available to them, the authors hypothesized that these teachers were more likely to try new strategies. With the drop in class skill levels, students in double-dose algebra had a higher rate of absenteeism and disciplinary problems than students in the above-norm group. Feelings of support and respect were largely unchanged and not statistically significant.

Upon further analysis, Nomi & Allensworth (2013) noted that test scores and pass rates were related to peer absenteeism and disciplinary issues. While this should have benefitted above-norm students, it was offset by the shift in class skill level as discussed earlier. For low-level students, enhanced pedagogy from their teacher was offset by the aforementioned absenteeism and behavior issues.

Booth et al. (2015) explored the effects of replacing traditional textbook problem sets with both correct and incorrect worked examples which students needed to analyze and explain. Traditionally, textbooks provide a host of problems that need to be solved. The 2007 IES Cognition Practice Guide recommended that teachers instead provide students with sets that consist of a worked-out example of a problem which is paired with a problem for the student to solve. However, both a lack of teacher resources and a lack of long-term studies made the effectiveness of this concept unclear. The research that had been conducted argued that an available worked example frees brain processing power which is typically used to recall information and instead applies it to better understanding the structures of the problem. When students worked through an incorrectly worked problem to correct it, it had positive effects on student learning. However, many of these studies were performed in a laboratory setting, and there was little real class data to support these findings. Counterarguments contested that worked examples narrow student understanding, particularly when applied to problems with variations. Some argued that worked examples should be phased out as students gain more mastery of a concept to ensure optimal learning.

To add clarity and depth to the existing research, the authors conducted two experiments. The first implemented worked examples in the Algebra 1 classroom over one unit, approximately 3-4 weeks in duration. This experiment was meant to provide evidence of the differences between laboratory setting studies and those done in the classroom. It also was used to determine whether students with different knowledge levels would benefit from worked examples differently than their peers. Experiment 2 also measured how different students responded to worked examples but on a much larger scale. The increase in data points allowed the authors to investigate influences from both prior knowledge and socioeconomic status.

Experiment 1 consisted of 56 Algebra 1 students in three classrooms from two districts in the mid-Atlantic region, five of which were ultimately excluded due to post-test absence. Half of the students in each class were randomly assigned to the treatment group and were given worked example assignments. The other students in the class were given assignments that contained similar problems but no worked examples. Students were all given an identical pretest and an identical post-test. Pretest scores and the distribution of high and low prior knowledge were nearly identical for both groups.

The results of Experiment 1 found that prior knowledge had a significant effect on the post-test scores. This makes sense, as students who come in with more prior knowledge would be expected to score higher on the post-test. When prior knowledge was not considered, the treatment group mean increased by four percent to 72% while the control group mean actually dropped by five percent to 64%. The finding of treatment group improvement agreed with laboratory findings, but the authors were surprised to find that worked example results did not seem to be affected by prior knowledge.

Booth et al. (2015) did notice a peculiar result, though, that provides grounds for further research. That is, low-achieving students in the treatment group performed exceptionally well on the post-test, improving their scores by 10%, while high-achieving students in the control group actually decreased by nine percent. This hints at the possibility that results may be driven by more than one mechanism.

Experiment 2 expanded to include 425 non-honors Algebra 1 students in 28 classes from five different districts, though 30 students were eventually removed due to lack of post-test results. Students were again randomly assigned to experimental and control groups. There was a slight difference in the number of low-socioeconomic students;the experimental group had 46% and the control group had 37%, respectively. The process for the experiment was essentially the same as the first; the main difference was that Experiment 2 included more potential topics than simply solving linear equations (which most studies including Experiment 1 use).

The results of Experiment 2 were much less conclusive. The only result of statistical significance was students with low prior knowledge tended to improve more from using worked examples than their peers with high prior knowledge. However, as a whole, examples did not have a significant impact on student performance. The authors hypothesized that this may have been due to the high rate of procedural misconceptions when solving linear equations; whereas, other topics may not be as prone to such problems which limits the effectiveness of worked examples.

Booth et al. (2015) encouraged further research to be done that includes standardized testing results and also research that studies the effects of using correct worked examples versus incorrect worked examples ("describe the error") on students with different levels of knowledge.

Studies have shown that despite a large increase in need for scientists, engineers, and technology specialists, the percentage of college students pursuing such degrees has remained stagnant or has decreased. Additionally, the average American student continues to rank well below peers from other top countries on international assessments. As the push for STEM advances, schools are looking for ways to increase student engagement and interest. One strategy involves supplementing traditional math courses with applied math courses, the two biggest of which are scientific research and engineering (SRE) and information technology (IT). Gottfried et al. (2014) studied the effectiveness of these applied courses. Specifically, the study "details which applied courses are most commonly taken by high school students; the extent to which high school students take applied courses alongside their traditional math courses; and whether applied courses are an effective means to boost student proficiency in math" (p. 2).

Gottfried et al. (2014) used the Education Longitudinal Survey (ELS: 2002), which administered questionnaires and assessments to 15,362 students from across the nation who were in 10th grade in the spring of 2002. Students were then reinterviewed and reassessed in the spring of 2004 and again in 2006, by which time most students had been out of school for two years. The ELS: 2002 also published student transcripts. For this study, 11,112 students met the criteria for participation. In order to stay true to the research goal, the authors controlled for socioeconomic factors, such as gender, race, and family income, as well as variables related to investments in schooling, such as expectations for college, extracurricular activity participation, and parent involvement.

Gottfried et al. (2014) found that 25.5% of the sample took an IT course as compared to 14.4% that had taken a SRE course. The authors noted that this was likely due to nine states that required computer science as a graduation requirement, while no states required SRE courses. Further analysis revealed that more than 1% of the sample took one or more of six IT courses; 11% of students took Computer Science 1 and 4.1% took Website Design. SRE courses, however, had much less commonality; only two courses met the 1% threshold. Those two courses were Drafting Fundamentals (3.7%) and Computer Assisted Design/Drafting, or CAD (3.5%).

Those that took SRE courses were less likely to be moving on to advanced math courses. Twenty-five percent of students taking SRE courses were below average on the course scale while the least likely group to take SRE courses (13.1%) were those in the advanced math course track. This indicated that students were taking SRE courses instead of advanced math, rather than as a complement to it. IT courses, on the other hand, appeared to have been taken throughout the levels of math curriculum and seemed to complement the traditional math courses.

Results for the final research question showed that overall the more advanced the student was in the math curriculum, the less impact applied STEM courses had compared to peers who did not take applied STEM courses; applied STEM courses had the biggest impact on students in the “below average” curriculum group and the smallest impact in the “advanced” curriculum group. This finding held true when SRE and IT were tested individually as opposed to as a package. However, when all was equal, students who took applied STEM scored higher than peers who did not take applied STEM courses. Since these courses seem to have benefitted “below average” students the most, Gottfried et al. (2014) encouraged their use as a way to keep those students engaged and supported in their traditional math courses.

In 2000, 35% of high school seniors scored as “Below Basic” on the math portion of the National Assessment of Educational Progress, while 83% performed lower than “Proficient.” Oklahoma scored even lower as evidenced by their 2004 state standardized testing, as 27% of students who had completed Algebra 1 scored in either the “Satisfactory” or “Advanced” category, as reported by the Oklahoma State Board of Education. As a way of potentially improving scores, some have suggested that content mastery is more profound when presented through familiar contexts such as agricultural education.

Parr et al. (2006) tested the notion that students in a contextualized, math-enhanced agricultural curriculum would develop a deeper, longer lasting understanding of math concepts than peers who remained in the traditional curriculum model. The authors posed two research questions:

RQ1) What were selected characteristics of students enrolled in, and instructors teaching, agricultural power and technology in the state of Oklahoma during the spring

> 2004 semester? and RQ2) Does a math-enhanced agricultural power and technology curriculum and aligned instructional approach affect a student's need for postsecondary math remediation? (p. 84)

Thirty-eight teachers participated in the study; 18 were randomly assigned to the experimental group and 20 were assigned to the control group. The experimental group teachers were paired with a math teacher to transform their curriculum into a math-enhanced curriculum. Through comparison of the current curriculum and state math standards, 17 lessons were developed that incorporated math concepts. During the trial period, agricultural teachers were expected to deliver these lessons without additional assistance from math peers.

To answer RQ1, students and staff were given a survey that included questions about gender, race, and grade level (students only). The results showed that 84.4% of students were male. Nearly 59% reported to be of European descent, while 25% reported to be Native American. About 66% of students were either juniors or seniors, and 26.4% were sophomores. When asked their age, 82.7% of students reported being between the ages of 16 and 18. Lastly, 72% of students self-reported grade point averages between 2.6 and 4.0. Data from the 38 participating teachers showed that 86.8% were male, 2.6% were female, and 10.8% chose not to answer. Racially, 73.7% reported having European descent and 15.8% reported Native American descent.

Prior to the study, students were administered the Terra Nova CAT Basic Battery examination which was used to ensure generally equivalent math aptitude across both groups. The results showed that there was no statistically significant difference in aptitude to start. At the end of the study, students took the ACCUPLACER, a test that is traditionally used to determine placement in postsecondary courses. As this study was part of a larger study and other tests were also being administered, only 125 of the 447 student participants completed the ACCUPLACER.

Parr et al. (2006) used a one-way analysis of variance (ANOVA) to assess the significance of the ACCUPLACER results. The control students had a mean score of 13.01 with a standard deviation of 3.24, while the experimental students had a mean score of 15.56 with a standard deviation of 2.92. This was statistically significant. To determine the practical significance of this result, the authors used Cohen's effect size. The effect size was .83, which is considered "large." This led the authors to conclude that a math-enhanced agricultural power and technology curriculum can positively affect the number of students needing remediation at the postsecondary level.

While the most commonly proposed solution to America's low math scores is to require more math courses in high school, Stone III et al. (2008) pointed out that data has shown this "more of the same" strategy has largely not been proven effective. The authors also noted that lack of engagement seems to be a contributing factor in students' lack of progress. Therefore, Stone III et al. (2008) proposed a solution: "Enhance career and technical education (CTE) courses with more rigorous, relevant mathematics" (p. 769). For example, students in a horticulture course could calculate the area of pots to determine how many would fit in the greenhouse, while construction courses can cover the Pythagorean Theorem and right triangle properties. These explicit connections, the authors argued, are crucial if students are to employ their math knowledge beyond the classroom.

Stone III et al. (2008) tested the hypothesis that students in these math-enhanced CTE courses would perform better than students in traditional CTE courses, seeing math as a tool in the same regard as a saw or hammer. Specifically, they posed the following research questions:

1) Does a math-enhanced CTE curriculum improve student math performance as measured by a traditional (TerraNova) test of math knowledge and skills?
2) Does an enhanced CTE curriculum decrease students' likelihood of requiring postsecondary math remediation, as demonstrated by improved scores on a college placement (ACCUPLACER) test?
3) Does a math-enhanced CTE curriculum improve student math performance as measured by an applied (WorkKeys) test of math knowledge and skills?
4) Does enhancing a CTE curriculum with mathematics reduce students' acquisition of occupational skills they will need for the workplace? (as measured by a variety of technical skills tests). (pp. 770-771)

Two hundred three teachers from across the nation volunteered to participate in the study, 95 of whom were randomly assigned to the experimental group. Due to missing data, however, only 137 teachers were included in the final analysis (59 experimental, 78 control). These 137 classrooms had a total of 595 participating students among them. CTE teachers were paired with a math teacher partner. The pair identified concepts in the curriculum that provided natural opportunities for math enhancement. CTE teachers were also given additional training on how to teach the identified math concepts. The study took place over a full school year and occurred in five course topics: business and marketing, auto technology, health technology, information technology, and agriculture.

At the beginning of the 2004-2005 school year, students were given a pretest via the TerraNova CTBS Survey to measure the equivalence of the experimental and control groups. Teachers were then instructed to conduct class as normal, with no reference to the change in curriculum. The math enhancements were required to be inserted naturally into the curriculum and not presented as a stand-alone topic. To ensure teachers were not teaching to a specific test, Stone III et al. (2008) employed three different standardized tests, the TerraNova CTBS Basic Battery, the ACCUPLACER, and the WorkKeys Applied Mathematics Assessment. Students were given 40 minutes to complete the pre- and posttest and the authors used the percentage of correct answers as the standard of measure. In addition to testing math skills, Stone III et al. (2008) tested students' technical skills using tests specific to each class' industry.

The results of the TerraNova posttest (RQ1) as well as the ACCUPLACER posttest (RQ2) revealed a significant improvement over pretest scores, with treatment being responsible for 42% and 35% of the score variance, respectively. The WorkKeys posttest (RQ3), however, did not show significant impact from treatment. The authors noted that this assessment involved more reading than the other tests, thus potentially, inadvertently assessing students' English comprehension as well. Finally, RQ4 asked about any potential negative effects of math-enhanced lessons on technical skills. Data from the technical skills assessments showed no negative effects, and one group of treatment students even showed significant improvement over their control peers. The authors, therefore, concluded that math enhancement within CTE courses should be seen as a low-risk policy that could reap positive results.

CTE teachers expressed a common challenge throughout the study; that is, students exhibited a surprising lack of readiness for the math concepts presented. In making math-enhanced lessons at the beginning of the study, teachers assumed a basic level of algebraic understanding. Once in the lessons, it often became apparent that many students did not understand foundational skills such as multiplication, ratios, and measurement. Thus, teachers needed to implement more remediation than expected, and frustration grew among both teachers and students. Despite this challenge, however, the CTE teachers largely indicated that they felt the Math-in-CTE model worked and was worthy of continued implementation.

CHAPTER III: DISCUSSION AND CONCLUSION

Historically, the American students have struggled in math. At the state level, the Minnesota Department of Education website stated only 45.4% of Minnesota 11th-grade students were considered “proficient” on the 2019 MCA assessment. Nationally, in 2019 the National Assessment of Educational Progress revealed that only 44% of 8th-grade students were “proficient” in math. Internationally, the United States has consistently scored below average in math (Bozick & Ingels, 2008; Gottfried et al., 2014; Kim et al., 2015; Ruff & Boes, 2014).

Education leaders and policymakers have spent three over three decades trying to find the answer to American students’ mathematics struggles. Currently, the majority of states require a college and career-readiness curriculum for graduation. Generally, this includes mandatory Algebra 2 as well as Algebra 1 in eighth grade. While some argue Algebra 2 completion increases the likelihood of college enrollment and higher career earnings, others argue that college-bound students take a more advanced course track that includes Algebra 2. In this argument, critics say that correlation does not equal causation. Furthermore, one must ask if making Algebra 2 mandatory for all ultimately decreases rigor as schools attempt to prevent an increase in failure and dropout rates among students who do not have the mathematical skills necessary to be successful in higher-level math.

According to Clotfelter et al. (2015), enrollment of eighth-grade students in Algebra 1 doubled from 1988 to 2007. This shift of Algebra 1 to eighth grade allows students to take a more advanced course track in high school without the need to "double up" on courses. Making Algebra 1 mandatory in middle school, thus eliminating "tracks," is also intended to close racial and socioeconomic achievement gaps (Crawley, 2018; Loveless, 2009; Nomi, 2012).

This literature review sought answers to the following questions: 1) Are the current United States mathematics curriculum policies producing the desired results and optimizing student achievement? and 2) What alternative policies have been attempted in the United States in order to improve student performance in mathematics? Thirty studies were reviewed for this literature and were separated into three main sections: 1) Anxiety and Feelings Toward Math As Potential Barriers to Learning (4 studies), 2) Analysis of Current Curriculum Shifts (20 studies), and 3) Non-Traditional Math Initiatives (6 studies).

Summary of Findings

In the first section, math anxiety tended to be significantly elevated during assessments as compared to instructional time. Small groups and test retake options through a Standards-Based Grading framework revealed a benefit to many students, though the effectiveness of test retakes was limited to students without communication apprehension. In my personal experience, this is a key finding, as many students feel apprehensive about approaching their math teacher and would fall into the communication apprehension classification.

Overwhelmingly, students' success in high school math is dependent on a combination of their level of prior knowledge as well as background factors such as race and socioeconomic status. Koon & Davis (2019) found fifth-grade achievement and whether or not a student is

Black to be significant predictors of future success. Bozick & Ingels (2008) echoed these findings in more general terms.

Studies that focused on Algebra 2's role in postsecondary achievement found that students who took Algebra 2 enrolled in college at a higher rate than those who did not, though most authors were quick to point out the potential influence of selection bias (Aughinbaugh, 2012; Gaertner et al., 2013; Kim et al., 2015). For example, Aughinbaugh (2012) found that college-bound students also took higher levels of science and foreign language as well. Therefore, one cannot attribute causality to Algebra 2 completion. Additionally, future income attributes higher wages to college degree attainment rather than Algebra 2 completion (Gaertner et al., 2013; Levine & Zimmerman, 1995).

While some studies found either a slight positive or negligible impact of mandatory eighth-grade Algebra 1 (Allensworth et al., 2009; Crawley, 2018; Domina et al., 2014; Gamoran & Hannigan, 2000; Sorensen et al., 2018), several studies found that this policy was actually hurting either all students or certain groups of students (Clotfelter et al., 2015; Domina et al., 2014; Liang et al., 2012; Loveless, 2008; Loveless, 2009; Nomi, 2012). In theory, while detracking should help to eliminate the racial and socioeconomic performance gaps, they tend to lower class skill levels (Nomi, 2012), decrease test scores, (Domina et al., 2014), and increase failure rates (Clotfelter et al., 2015). Liang et al. (2012) found that students in the middle fared better in the long term by waiting until ninth grade to take Algebra 1.

The concept of "Double-Dose" Algebra 1, the short-lived policy implemented by Chicago Public Schools, showed slightly positive results for those slightly below average. For all other students, though, the initiative had a negative effect on performance (Nomi & Allensworth, 2009; 2013). Booth et al. (2015)'s study on worked examples proved beneficial for low-level learners, but it did not have significant influence elsewhere. Math-enhanced curriculum, however, did see positive results across the board. Incorporating math into Ag classes (Parr et al., 2006), CTE courses (Stone et al., 2008), and applied STEM courses (Gottfried et al., 2014) proved beneficial to test scores as well as keeping low-level students engaged in math content.

Professional Application

On an international level, Son & Senk (2010) offered a view into the South Korean curriculum focus on multi-step problems and problems involving thought and reasoning. American curriculum, on the other hand, is much more process-driven. Perhaps problems involving thought and reason, though potentially more challenging on the surface, could encourage students to invest more in the content. This also ties into the effectiveness of a math-enhanced curriculum, whereas students see their math learning come to life rather than a set of book problems to finish. This doesn't need to only happen in a CTE or applied STEM classroom; in fact, math teachers need to continue to be intentional about finding real-world applications whenever possible.

While teachers have little-to-no control over state and national policies, knowledge of the data presented in this literature review can potentially help influence policy at the school level. First, could collaboration between the math and CTE departments be regularly infused without too much added to teachers' already-full plates? In other words, could the math-enhanced curriculum studied by Gottfried et al. (2014), Parr et al. (2006), and Stone et al. (2008) be expanded to an entire year? Second, teachers and counselors need to be aware of the implications of scheduling students in certain math classes. For example, if given the option

Liang et al. (2012) found that students who score near the mean on the previous year's standardized test benefit by waiting until ninth grade to take Algebra 1. Three, another glaring component found throughout this research was the importance of a strong foundation. Students who entered middle school and high school with a higher-level of math knowledge were ultimately more successful (Bozick & Ingels, 2008; Koon & Davis, 2019; Sciarra, 2010). Thus, utmost importance ought to be placed on giving elementary teachers tools and resources that optimize student performance.

Limitations of the Research

There are three main limitations to this literature review. First, while some of the studies were national and therefore included Minnesotan students, none were specifically focused on Minnesota education. By analyzing research from across the country, regional culture differences were not taken into consideration. Studies from the east coast, for example, may have had different results if recreated in the upper Midwest due to cultural differences. Historically, Minnesota has prided itself on providing an education that is toward the top of United States rankings. This culture of pride in education may also affect study results.

The second limitation is the lack of studies post-Common Core implementation. While several of the studies took place after Common Core's 2009 implementation, it is fair to assume that there has not yet been enough time for the results to be heavily impacted by the curriculum change. On the other hand, if Common Core did have an influence on the results, no studies included in this literature review took that into consideration.

Last, selection bias was by far the most commonly noted limitation in the reviewed studies. While many worked to eliminate as much selection bias as possible, it may not have been possible to eliminate all of it. To illustrate, a student with career aspirations requiring a 4-year degree will almost always tend to take more challenging courses than a student who plans to enter a manual labor job immediately after graduation. In cases such as this, Algebra 2 and other similar courses cannot claim the credit for college enrollment and potential earnings differences.

Implications for Future Research

As mentioned above, it would be interesting to see these studies replicated in Minnesota. This would possibly reveal cultural differences, as well as curriculum differences. Since Minnesota's math standards were last updated in 2007, would the results differ when compared to states that have fully implemented Common Core standards? Do Minnesota's relatively outdated standards affect Minnesota students' performance on national and international standardized tests?

Another area of future research I would like to see is an analysis of the frequency of each state standard in the main job industries. For example, which professions regularly use Minnesota math standard 9.2.1.7, which states "Understand the concept of an asymptote and identify asymptotes for exponential functions and reciprocals of linear functions, using symbolic and graphical methods"? Are there standards that can be considered "dead" due to lack of use in the real world? Additionally, there has been a significant change in technology since 2007. Since technology has so quickly become an integral part of our lives, do we still need to spend time teaching certain concepts? For example, instead of having students memorize the quadratic formula, would it be a better use of time to show them an online quadratic formula calculator and then use the remaining time applying it to real-world scenarios?

As the findings from these reviewed studies revealed a minimal positive effect of Algebra 2 along with positive effects of math-enhanced curriculum, would students be better served if math curriculum requirements instead included math-enhanced CTE, STEM, or Finance courses? Could this application-based curriculum also help to reduce the overwhelmingly negative view of math among Americans of all ages? If students had more positive attitudes toward math, would their children achieve more as a result of having a more positive support structure?

Conclusion

As it stands today, our math system is broken and ineffective. Despite more than three decades of trying to "do more to achieve more," the United States has not been able to significantly raise its scores. Furthermore, negativity and anxiety surrounding math continue to worsen. The data shows that what we are doing isn't working for the majority of students, typically those not in the "advanced" group. This study revealed positive impacts on learning when a math-enhanced curriculum was implemented in elective courses. Students who take these courses are typically on the lower end of math achievement, and this initiative seems to be effective in engaging those students in math while also preparing them for technical careers.

When my students continue to try the same methods despite repeated failures, I tell them, "Don't be a junebug." This is a reference to a junebug's incessant attempts to fly through a window; the constant banging gets them nowhere. "If your method isn't working," I tell my students, "you need to try a different method." For over 30 years, American math policy has been stuck hoping in vain for a better result. Perhaps, it is time to try something different. America, "Don't be a junebug."

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