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Factors Impacting Mathematically Literate Students

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FACTORS IMPACTING MATHEMATICALLY LITERATE STUDENTS

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Abstract

The question analyzed within this literature review is: what are common traits of mathematics learners and how can we utilize this information to promote mathematical literacy in the classroom? Mathematical literacy is a large factor in this development because it involves a person's ability to utilize mathematics in a realistic and practical manner, therefore inhibiting or enhancing the STEM field. The research concluded that mathematical literacy is not affected by a single factor, but is a multifaceted concept that requires support in many areas of academics. It is recommended that students be taught in their primary language and in an interdisciplinary way utilizing specific content vocabulary and language strategies.

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CHAPTER I: INTRODUCTION

Mathematics, as a discipline, was distinguished in ancient history. The original concepts of mathematics lie in the number sense of quantity, such as, "you have *more*," "I do not have *any*," "humans have a *number* of toes." Humans have been telling time and creating calendars for twenty-thousand years, expressing their need for a quantifiable representation of their daily lives and making time a distinct component of early mathematics. It is in these generalized observations that humans started recognizing there was a way to describe these concepts.

It is by increasing the level of practical applications of mathematics that one can find the use of a more recent term *mathematical literacy*, coined for its use in conjunction with language literacy (Boyer & Merzbach, 1991). For this literature review, the definition of mathematical literacy is: being literate in the language of mathematics (Ippolito, Dobbs, & Charner-Laird, 2017). This concept can be connected with *numeracy* which is defined as the ability to understand and work with numbers. For example, a person can be literate in mathematics if they understand the vocabulary and meaning behind concepts when working within the constructs of numbers. At times, observations have been found when referencing statistical understandings of mathematics. *Quantitative Literacy* and mathematical literacy are interchangeable in these instances. Quantitative Literacy "requires one to understand the nature of mathematics and its role in scientific inquiry and technological progress" (Steen, 2001, p. 8).

Historical Context

While mathematical literacy is a recently developed term, the concept itself goes back in history. For example, Benjamin Franklin and Thomas Jefferson promoted an early skill called *numeracy* to unify the people of colonial America and give them a necessary level of knowledge to participate as productive citizens (Steen, 2001). From the time of America's birth and into the early decades of its life as a sovereign nation, numeracy rates of the population were extremely low. Interestingly, with the changing of centuries, mathematicians started noticing an expectation of ordinary citizens that included being quantitatively literate. The term *political arithmetic* arose with the democratic representation alerting the public that this political system required counting and a census (Cohen, 2003). The United States Constitution relied on three ideas of quantification, including a representative government (dependent on population size); a census and direct taxes (Cohen, 2003).

The first instance where the term *statistic* was found within an American dictionary was in 1803, prefacing a quantification shift (Cohen, 2003). It was around this time period that the monetary methods of the country changed and therefore required less sophisticated mathematical knowledge. A piece of writing from this time frame stated, "bad governments prefer complicated money and innumerate citizens" and it was for this reason that Thomas Jefferson drafted the monetary change (Cohen, 2003). Unfortunately, even with simplifying the system, a large portion of the country were still challenged since the extent of its math skills were acquired from home and many students did not receive more than an age ten education.

Coinciding with this process was an educational reform. Schools were making a shift from *traditional-style* mathematics to *college-style*. In the traditional-style of education, students were exposed to simpler level arithmetic and postponed the introduction to complex subtopics such as geometry and statistics to university. Conversely, college-style methods equipped students with a large array of content knowledge at the surface level. Over time, more diverse subsections of the population were able to attend school and for a longer duration. It was found that the curriculum educators used was not aligning with this change, and it was not until the antebellum era that economic numeracy and statistics showed up in public written works (Cohen, 2003; Steen, 2001).

Leading into the twenty-first century, people were required to have "content knowledge, but also required skills including critical thinking, problem-solving, creativity, innovation, communication, collaboration, flexibility, adaptability, initiative, self-diversion, social, cross-culture, productivity and accountability, leadership and responsibility, and information literacy" (Rizki & Priatna, 2019, p. 1). Thus, proving that having a term to encompass all these mathematical needs became essential. Researchers found that not only having a category of mathematical literacy was important, but also termed the fundamental process of making connections with abstract and practical math as *mathematization* (Rizki & Priatna, 2019). More recently the National Council for Teachers of Mathematics released an article stating their mathematical standards, creating "considerable public debate about the goals of education and about the relation of mathematics to these broader goals", leading to the division "into three components: prose, document, and quantitative literacy" (NCTM, 2000, p. 3). That is the knowledge and skills necessary to perform spoken, written, and computational tasks (NCTM, 2000).

Conversations about mathematical literacy have occurred increasingly over the past two decades and have led to a paradigm shift from not only teaching mathematical literacy but reading in the mathematics classroom (Beaudine, 2018). That poses the question, how should teaching literacy in the mathematics classroom look? The Organization for Economic Co-Operation and Development (OECD) states that "we must attend to three components (1) writing, (2) discussion, and (3) reading" (OECD, 2013, p. 59). While this may be a good start, students need more exposure to different methods of practicing literacy to see how everything is intertwined.

The Relevance of Mathematical Literacy

In the United States, people have begun to “recognize the critical role of an everyday understanding and appreciation of mathematics as an important characteristic of a well-formed citizen and productive worker” (Sasanguie, De Smedt, Defever, & Reynvoet, 2012, p. 344-357). Unfortunately, the United States literacy levels are lower than thirty-six other developed countries in the area of mathematics (Sasanguie et al., 2012). It would be beneficial to look at what other countries such as Singapore and Switzerland are doing to have such high scores along with examining our students who are excelling to identify commonalities. In a country that prides itself on Science, Technology, Engineering, and Mathematics (STEM) knowledge in a highly developed world, it is apparent that something is lacking with United States mathematics instruction.

This research topic provides a unique opportunity to improve mathematics instruction and assessment strategies through a focus on questions such as: what are other countries doing to promote mathematical literacy? What courses have the most significant impact on quantitative literacy? What is the relationship between numeracy and literacy? Recently, a survey showed that employers are concerned with students' quantitative literacy skills, recognizing the range of high-level skills required for career tasks (Rhodes, 2010). The desire for quantitative literacy could benefit students now and in their futures.

Brief Overview

Throughout this thesis, the research question analyzed is what are common traits of tenacious mathematics students, and how can these commonalities be used to aid the teaching community in the promotion of mathematical literacy? The structure of this review is to present the current research in this order: what is mathematical literacy, why do we need to increase mathematical literacy, what traits do students need to be mathematically literate, and how can one teach for mathematical literacy? Following the literature review will be a summary and discussion of the limitations and implications of current and future research.

Defining mathematical literacy and its necessary research parameters is essential when completing a literature review to ensure accurate and pertinent information. Mathematical literacy is analyzed through relevant research, including questions posed that are relevant, recent (past two decades), published in a peer-reviewed journal, and focused on teacher education in the United States (Kilpatrick, 2001). While the focus of mathematics education should primarily be researched using one's own country of origin, it is naive to think that the United States is the only country that can offer relevant information regarding mathematical literacy. Therefore, this review will analyze studies from a variety of countries.

If the findings of this literature review prove to be advantageous, it could ultimately be utilized for the development of new forms of curriculum. The field of education would gain from increased mathematical literacy because more students would have a higher level of critical thinking and problem-solving skills. Educational shifts involving increased mathematical literacy could be a tremendous force that changes the dynamics of mathematics for many years to come.

CHAPTER II: LITERATURE REVIEW

Literature Search Procedures

Searches conducted of Educator's Reference Complete, Expanded Academic ASAP, Education Journals, ERIC, Academic Search Premier, and EBSCO MegaFILE for publications from 1991-2019 were used to identify literature for this review. This list was narrowed by only reviewing published empirical studies from peer-reviewed journals that focused on math, literacy, and professional development literacy (in math content area) that addressed the guiding questions. The keywords used in these searches included: "content literacy strategies", "math", "math reading comprehension", "professional development math literacy", "professional development content literacy", "math literacy", and "quantitative literacy". The structure of this chapter is to review the literature on mathematical literacy on the following topics; a definition of math literacy, the need for mathematical literacy, factors impacting mathematical learners, and teaching mathematical literacy.

What is Mathematical Literacy?

Kilpatrick (2001) sought to provide assistance to educators in developing the term "mathematical proficiency" by completing a literature review involving sixteen people with expertise in classroom practice, the mathematical sciences, research in cognitive science, business, and research in mathematics education. Following this compilation, Kilpatrick (2001) used the phrase, mathematical proficiency, which redirects the extreme goals of mathematics to focus on combining literature *and* practice. In an attempt to remove obscurity, the committee that completed the research determined what it means to be successful in mathematics, what areas are essential in kindergarten through eighth grade for continued learning, and its role in influencing programs and policies (Kilpatrick, 2001). Admittedly, though this study was useful in determining the effectiveness of research already completed in the area of mathematical literacy, it did little to help one understand mathematical literacy itself.

In response, Yore, Pim, and Tuan (2007) contrasted the ideas of numeracy and proficiency to research the idea of mathematical literacy. They identified that while numeracy indicates the level of understanding necessary in a classroom setting, math literacy has less to do with the school curricula and more to do with a productive citizen approach to mathematics. They suggested that mathematical literacy requires a fundamental sense (reading and writing in the discipline) and derived sense (content knowledge in the discipline) (Yore, Pim & Tuan, 2007). Mathematical literacy requires mathematical proficiency to develop a comprehensive understanding.

Overall, determining how to teach for mathematical literacy would be beneficial; however, without consensus regarding a definition, that is unlikely (Haara et al., 2017). Utilizing the definition provided by the OECD countries could be beneficial.

The Need for Mathematical Literacy

It is known that the United States prides itself on being a world power. To maintain that status, scientists and engineers need to continue to develop new products and make pivotal discoveries, therefore those who are mathematically literate are needed. Helping the current and future generations to increase their mathematical proficiency and literacy would greatly benefit the expansion of STEM careers. Researchers identified that mathematical literacy is important to develop problem-solving skills and for the United States to compete in the twenty-first century (Rizki & Priatna, 2019). Unfortunately, there is a significant stigma of mathematics being termed tedious or too challenging which creates barriers for students in the classroom.

Similarly, Wilkins (2000) searched for specific benchmarks of math content knowledge, reasoning, societal impact and utility, nature/history of math, and mathematical disposition. He determined that a quantitatively literate person possesses the following: mathematical reasoning skills, real-life applications of mathematics, a historical overview, a positive attitude towards and knowledge of mathematical content (Wilkins, 2000). While 76% of American students documented a feeling of confidence in mathematics, more than students from any other country, and this is telling because it was suggested this was due to U.S. parents having a higher tendency to tell children that they can do anything they set their mind to, which may lead to a false sense of confidence versus the necessary skills that Wilkins (2000) mentions above for quantitative literacy. Additionally, 62% of the United States' participants expressed enjoying mathematics to some extent which is higher than twelve countries (Wilkins, 2000). One possible reason for the positive attitude towards mathematics but the disappointing ranking scores are attributed to the United States educational system, which allows low achieving mathematics students to repeatedly take courses that are largely filled with review (Wilkins, 2000). Although positive attitude is listed as necessary for quantitative literacy and remedial courses could be beneficial if they actually led to attaining higher skills; in reality, this does not appear to be occurring in U.S. schools. Hence, this study recommended cooperative learning to support an inquiry process to help foster the creation of one's understanding and integration of mathematical content areas for how one can best help future generations (Wilkins, 2000). Information such as this upholds the findings that mathematics instruction may need to change.

A literature review completed by Haara, Bolstad, and Jenssen (2017) was focused on identifying empirical projects of mathematical comprehension and mathematical literacy in schools. The focus participants in these studies were students in primary and lower secondary schools. The authors found that many of the research articles on math literacy did not emphasize qualitative approaches the mathematical literacy-enhancing classroom (Haara et al., 2017). As a result, teachers do not know what to prioritize. Consequently, the workforce is affected, resulting in a lower population of contributing citizens with an ability to apply mathematical concepts on an everyday basis (Wicklein & Schell, 1995). Therefore, the ranking of the United States as a world power is jeopardized.

During the Cold War, the United States was at the forefront in a STEM race with other world powers to increase their global standing. Recent research shows that the current ranking compared to other developed countries is declining (PISA, 2018; Wilkins, 2000). The Programme for International Student Assessment (PISA) used an assessment-based, statistical analysis in seventy-three countries to determine how fifteen-year-old students of the United States compared to those of seventy-nine other countries (PISA, 2018). They found that 36 education systems had higher average math scores than the United States including Canada, Australia, Sweden, and Singapore (PISA, 2018).

The study also found that while the United States had a similar level of students with a low proficiency level (lower than two), the high achieving student population is only 6% compared to an average of 11% found by the Organisation for Economic Co-Operation and Development (PISA, 2018). Limitations of this research included a lack of suggestions to change the rankings in favor of the United States on a global scale. As such, it is hard to say what is a definitive contributing factor to these percentages of mathematical proficiency without further context.

Factors Impacting Mathematics Learners

Amidst the content area of mathematics, many factors would be deemed beneficial while others are detrimental. This section aims to identify the positive factors that impact mathematics learners in hopes of being able to instill those factors in every student. From a student perspective, factors include language and learning differences (including math anxiety), age- and gender-related differences, and student self-efficacy beliefs. Following the student factors, educator characteristics and barriers will be examined to distinguish strategies to best help promote and teach mathematical literacy.

Learning Factors

The first factor acknowledged is an inhibitor of mathematics that students generalized as learning disorders. These conditions impede a student's ability to show proficiency. Similarly, fear, poor attitude, misunderstanding practical applications, class sizes, scarcity of resources, and an inability to understand math applications are barriers to students' understandings (Ankita & Richa, 2017). Due to the barriers that can impact students with math learning disorders, it is important to identify students and provide supports (when needed) promptly.

Previously, preliminary research of student-related factors focused on the incidence of a Math Learning Disorder (MLD) among school-aged children in Rochester, Minnesota. Barbaresi et al.'s (2005) purpose was to investigate the prevalence of math learning disorders singularly or affiliated with a reading disorder. The population studied comprised of children born from 1976 to 1982 who remained in the Rochester area after age five. Through this retrospective approach, researchers identified markers of learning disorders by using school and medical data. This study found that by age 19, about 10% of the students had a Math Learning Disorder diagnosis (46% a reading disorder alongside an MLD), and the boys represented a larger proportion of diagnoses

(Barbaresi et al., 2005). Barbaresi et al. (2005) suggested that this was a large number of instances for a population and therefore recommends early identification of these students using a proactive approach. It is imperative to meet the needs of students with an MLD by offering supports and accommodations when necessary.

Another factor students face relates to family barriers such as a lack of encouragement or negative outlooks. Parental education and attitudes also contribute immensely to a student's interest or attitude towards a subject and without the support of their parents and family, students have a lower passing rate and therefore are less successful moving forward in mathematics (Ankita & Richa, 2017). Additionally, if a family member is unaware or uninterested in the subject of mathematics, they are unable to create a home environment supporting math literacy (Ankita & Richa, 2017). Provided that a supportive family member monitors the student, one might see reductions in the presence of this factor.

Equally concerning is the school-related barriers that come from an infinite number of sources. Often overlooked in this subsection are the school environment and infrastructure. Students are affected by environmental features such as harsh lighting and consistently unpredictable temperatures (Ankita & Richa, 2017). Environmental factors can cause students to become anxious, stressed, tired, or uncomfortable which makes it hard to focus on their learning. Unfortunately, the current school culture also requires the acknowledgment of physical safety in order to have a successful learning experience. It is disheartening to know that educators train students from a young age to take part in lockdown drills and the media makes available the content covering school shootings and other crises across the country (Ankita & Richa, 2017). Current students are more prepared for contact with a gunman than most adults, and this lack of feeling safe can contribute to a student's inability to focus on intellectual tasks (Ankita & Richa, 2017).

There is a vast disservice at play when considering the unidentifiable learning disorders and barriers present in all populations of children. Students cannot possess an equal opportunity if their learning abilities are not easily understood and protected. By monitoring students with mathematical learning disorders, it allows one to find students that excelled regardless of a learning disorder and those who require additional help which may include students suffering from language barriers.

Language Factors

Is it reasonable to expect a student to learn the language of mathematics if they are not fluent in the language it is being taught? Henry, Nistor, and Baltes (2016) sought to determine the proficiency of English language learners and the predictive nature of their capacity by directing attention to the statistic that 23% of children in the United States are immigrants. These authors studied 1,200 students from South Florida Elementary through convenience sampling to take part in a quantitative analysis that considered gender, socioeconomic status, and grade-level in unison with English proficiency to determine predictions of math knowledge levels (Henry et al., 2016). Henry et al. (2016) found that there was no significant predictive impact on math scores regarding gender or socioeconomic status. However, grade-level proved to be pivotal with third grade having the most impact.

Henry et al.'s (2016) research also confirmed that the more literate a student was in the English language, the higher their mathematics scores were. This confirmed that English proficiency precedes mathematical proficiency, when the student is taught in English (Henry et al., 2016). Henry et al. (2016) only studied the English proficiency of Spanish-speaking students and did not include children of other native languages. The current study did not take into consideration the quality of the teacher as a factor (Henry et al., 2016). Additionally, a study from Germany showed that understanding language and language proficiency has a more significant influence on achievement in mathematics content than reading (Cramer, 2013). Cramer (2013) identified that mastering a new language includes two different proficiency levels. Development of basic interpersonal relationship skills requires two years of exposure and cognitive academic language proficiency takes five years of exposure (Cramer, 2013).

Moschkovich offered a similar viewpoint on assisting English language learners in mathematical literacy. He references that academic language in mathematics requires the use of three semiotic systems including natural language, math symbol systems, and visual displays (Moschkovich, 2015). Moschkovich (2015) identified that the separation of mathematics and language is hugely detrimental to English language learners because they may follow the mathematics skills but then cannot express their understanding of it through language. Alternatively, Moschkovich (2015) suggests that the English used in mathematics is not the same English used to speak fluently and therefore provides an opportunity for English language learners to feel included and become active participants in the mathematics classroom. Conceptual understanding in mathematics is unique because it represents finding meaning in mathematical contexts (Moschkovich, 2015). Moschkovich (2015) suggests that when analyzing and creating curricula, we need to remember that academic language in mathematics includes proficiency, practice, and discourse. For all students, language assessment should be based on comprehension rather than complexity. When this is not the case, assessments test students on linguistic complexity rather than mathematical content (Moschkovich, 2015). One could attribute this, in part, to the achievement gap for English language learners in the current educational system (Moschkovich, 2015). Education must give this population of students an equal opportunity for high-level mathematical reasoning to promote critical thinking skills, regardless of English proficiency level, to further develop both math and English competencies (Moschkovich, 2015). Therefore, treating language as a deficit should not occur and instruction should include multiple representations of the math content to give these students different perspectives to view the assigned material (Moschkovich, 2015). Current educators need to prepare to work with English language learners by considering how each task should look and what students should gain from the activity (Moschkovich, 2015). Furthermore, the preparation placed on the presentation of a lesson can increase the comprehension of English language learners and reduce stress for all students.

Math Anxiety

The prevalence of mental health conditions is rapidly intensifying across the country and it unsympathetically suppresses students. Math anxiety can be especially problematic, which contributes to adverse emotional reactions to math or the prospect of doing math (Maloney & Beilock, 2012; Punaro & Reeves, 2012). It can be depicted through poor achievement and an increase in behavioral problems at school, including avoidance, less time studying, and lower-class engagement (Punaro & Reeves, 2012). Elevated levels of worry when completing mathematical problems are unique to this form of anxiety and is appalling in comparison to content specific anxiety in other subjects (Punaro & Reeves, 2012). This high level of worry impedes working memory capacity and is associated with poor spatial processing, dot enumeration, and symbolic number comparison (Punaro & Reeves, 2012). It leads to high levels of anxiety which corresponds negatively to mathematical success (Punaro & Reeves, 2012). In other words, as the task difficulty increases, so does the math anxiety (Punaro & Reeves, 2012).

Similarly, a 2015 study completed on 35 students found high levels of anxiety within the Serbian student body. Students who had an interest in mathematics and correctly applied knowledge and learning strategies to the content were not any less anxious (Radisic, Videnovic, & Baucal, 2015). In turn, we see an inverse relationship between the more a student identified as anxious and mathematical achievement rates (Radisic et al., 2015).

Moreover, mathematical anxiety is thought to emerge in pre-adolescence and peak in the early high school years (Punaro & Reeves, 2012). Punaro and Reeves (2012) proved that worry about mathematics starts as young as nine years old and is associated with a distinct pattern of neural activity in the right amygdala, which is the portion of the brain responsible for regulating emotions and completing numerical computations (Maloney & Beilock, 2012). This hyperactivity also causes increased negative emotions, which leads to reduced activity in the region utilized for the working memory and numerical processing (Maloney & Beilock, 2012).

When challenges arise while working with the building blocks of mathematics such as addition, subtraction, and number sense, educators are able to identify math anxiety due to the direct relationship they share. To identify students considered at-risk for math anxiety, these authors recommend bolstering necessary mathematical skills at an early age because math anxiety is associated with deficits in one or more of the fundamental building blocks (Maloney & Beilock, 2012).

Gender- and Age-Related Factors

According to research completed by Piaget, cognitive development occurs in four stages: sensorimotor (birth to two), preoperational (two to seven), concrete operational (seven to twelve), and formal operational (adolescence to adulthood) (McLeod, 2018). Piaget's findings indicate there are developmental changes in the brain at certain ages that make specific ways of thinking possible or probable. Besides the age-related differences, there is always a humanistic curiosity of the differences in ability level of males and females, and the cause for these dissimilarities is currently unknown.

Age-related factors. Mathematics, as a science, is something that develops continuously over one's lifetime. It is for this reason that specific benchmarks associated with "normal" or "typical" age development are linked, as seen in Piaget's research. It is no surprise then that age-related differences can impact a student's ability to learn and therefore determine teaching strategies for an educator (Sasanguie et al., 2012).

Sasanguie et al. (2012) set out to identify age-related differences in number processing and mathematical achievement. Their study consisted of 118 participants (ages five through eight) partaking in curriculum-based standardized tests (Sasanguie et al., 2012). In comparison, literature by Setiawati, Herman, and Jupri (2017) highlighted age-related differences between twelve- and thirteen-year-old students. Their objective was to discover the cause of repeated errors on problems directed towards mathematical reasoning (Setiawati et al., 2017). The authors' study on 61 students from Indonesia found that there were arithmetic errors indicative of difficulties understanding algebraic concepts. Consequently, students were found to have trouble interpreting the meaning behind variables and the unknown. Additionally, the complexity of students' mathematical precision and calculations increased (Sasanguie et al., 2012). These findings suggest that the learning process starts with contextual learning so that students can understand the concept and invite others to participate. Issues arise because students do use previous knowledge, but it is not known yet how accurately this knowledge is applied (Setiawati et al., 2017).

The work completed by these researchers is noteworthy to every level of educator due to the restrictions and guidelines they provide about mathematical capabilities at these ages (Sasanguie et al., 2012; Setiawati et al., 2017). If time is taken to align the curriculum with the capabilities of each developmental stage, then best practices would be best utilized for each grade level. Without an understanding of this research on age, a student's performance could be altered moving forward (Sasanguie et al., 2012; Setiawati et al., 2017).

Gender-related factors. Previous research has shown that male and female ability levels are too alike to denote any statistically significant difference but some patterns for approaching mathematical content differently can be found. Lailiyah (2017) conducted research on this topic and looked at the following indicators: (1) making mathematical models, (2) writing answers logically, (3) utilizing models to find answers, (4) choosing/comparing strategies, (5) linking information with experiences, (6) and manipulating formulas (p. 4). Four teachers were chosen to complete a qualitative study where students were given a lesson followed by a test. The only difference found between genders was the indicator linking information with experiences. The author suggested that there is a notable difference between genders when building ideas, and the male subject was more detail-oriented than the female subject (Lailiyah, 2017).

Interestingly, this study took place in Indonesia and it reported that the students found it challenging to estimate answers and judge the reasonableness of solutions which is similar to what is reported on the United States mathematical literacy studies (Lailiyah, 2017; PISA, 2018; Wilkins, 2000;). This study was conducted using specific parameters and did not include a large discussion. As a result, the lack of details makes it challenging to recognize commonalities between this study and others on the topic of gender differences in mathematical literacy. More research would need to be conducted to determine validity.

While research supports that male and female students have equal computational abilities, discouragingly, math is still stereotyped as a male domain (Lindberg, Hyde, Petersen, & Linn, 2010). For example, Lindberg et al. (2010) found that parents believe sons have higher IQs than daughters, and girls report lower competence scores even when actual scores show only a marginal difference. Their research also suggests that women's performance level decrease when stereotypes are present, and in countries where gender is identified as equal, this gap in gender performance decreases (Lindberg et al., 2010).

If there are not statistically significant mathematical differences among gender, it would lead one to believe that the stereotypes themselves are what holds merit in the performance of students. This indicates that educators must foster self-efficacy beliefs to make considerable strides in increasing mathematical literacy (Lindberg et al., 2010).

Self-Efficacy

Self-efficacy refers to an individual's belief in his or her capacity to execute behaviors necessary to produce specific levels of performance (Bandura, 1977, 1986, 1997). One can consider many variables when determining what makes up self-efficacy in mathematics, including biological factors, learning styles, interests, and attitudes (educator self-efficacy is described in the subsection on teaching mathematical literacy).

Lubinski and Benbow (2006) wanted to see if there were biological factors such as age or gender that led to increased levels of self-efficacy. They aimed to understand mathematically talented youth by identifying their developmental pathways and talent acquisition within education. Students in this longitudinal research study were between the ages of 12 and 13 and in the seventh or eighth grade (Lubinski & Benbow, 2006). All students chosen earned scores within the top 3% on their achievement tests and were therefore labeled as mathematically talented (Lubinski & Benbow, 2006). They found that females displayed higher levels of verbal ability than males, leading them to be a more significant section of the high-level workforce because of their verbal-linguistic abilities (Lubinski & Benbow, 2006). Cleary and Chen (2009) set out to examine whether the importance of the processes relative to student math achievement varies across the early middle school years and the level of the math course. The participants were 2,100 middle school students from upper-middle-class families in the Northeastern United States. The authors found that students in seventh-grade exhibited more behaviors such as avoidance and forgetfulness than those of the other grade-levels (Cleary & Chen, 2009). Girls were likely to exhibit more frequent uses of positive strategies when studying mathematics and also presented higher levels of interest in the mathematics classroom (Cleary & Chen, 2009). These differences align with research completed by developmental researchers, but they have also shown that whereas gender differences in language arts often continue into adolescence, the gap observed in math is decreasing, with many studies showing genders to be equal in interest and instrumentality (Cleary & Chen, 2009). In addition, students in advanced courses reported higher standards than students enrolled in regular level math courses (Cleary & Chen, 2009).

Ozgen and Bindak (2011) set out to determine if sex, class, school type, math degree, education of parents, or importance given to the subject of math caused the most change in self-efficacy. Using a descriptive survey method, Ozgen and Bindak (2011) found males to have a more positive self-efficacy belief, and ninth-grade students had even firmer beliefs, but cultural situations profoundly influenced self-efficacy. Ninth-grade students had superior self-efficacy beliefs compared to their twelfth-grade counterparts and this is not representative of related research (Ozgen & Bindak, 2011). Researchers attributed this to university entrance exams and the fluidity of self-efficacy values, giving snapshots of a student’s beliefs (Ozgen & Bindak, 2011). The authors recommend using the strategies of creating learning goals, giving extensive feedback, individualized learning, encouragement, and useful, successful student examples in the classroom to help increase mathematical literacy self-efficacy beliefs (Ozgen & Bindak, 2011).

Ozgen (2013) also found observation and perception of knowledge to hold meaning in the self-efficacy beliefs among high school students in Turkey. The findings of this research showed that success is a predictor of self-efficacy, and therefore positively correlated (Ozgen, 2013). Ozgen (2013) studied ten students, each from grade levels 9 through 12. Eighty percent of students said there is a connection between math and the real world, and students with higher rates of self-efficacy said the real-world application makes math easier (Ozgen, 2013). Ozgen (2013) stated that students felt that their teachers did not present enough real-world connections, and therefore the students were unaware of when they would ever use the content they were learning (Ozgen, 2013). He speculated that students could not identify mathematics as anything other than a school subject (Ozgen, 2013). It is important for teachers to emphasize the relevance of mathematics so students can conceptualize the reasonableness of their findings and develop confidence in their understandings (Ozgen, 2013).

Research suggests that math literacy education requires non-traditional (problem-solving) teaching methods to mimic these practical real-world applications. Samuelsson's (2008) goal was to examine the effect of traditional versus non-traditional methods on the first five years of a child's education (105 students in pre-school compared to a national test administered at age five). Results indicated students' progress in conceptual understanding, strategic competence, and adaptive reasoning is significantly better when teachers use a problem-based curriculum; this could be due to engagement and motivation (Samuelsson, 2008). On the other hand, when teachers include an emphasis on academics, whole-class instruction, question-answer practices, increased teacher expectations and extensive feedback (traditional method); achievement is given an opportunity to improve especially in relation to self-efficacy (Samuelsson, 2008). If the goal is a holistic awareness of mathematical literacy, both methods, and approaches should be utilized when appropriate.

In relation to self-efficacy, Bryan and Bryan (1991) studied the effect of positive mood on the analytical performance of African American and Hispanic students (with learning disabilities). Results reveal students who experienced happy thoughts before completing mathematical calculations performed at a significantly higher rate of accuracy (Bryan et al., 1991). It is suggested that students are potentially able to shape their own positive outlook, therefore, initiating the productive process (Bryan et al., 1991). While positive mood is a student-centered factor, the onus is likely on the teacher. In times of low mood, they must harness the power of relationships to shift to a more positive attitude, thereby supporting student self-efficacy regulation.

While confidence is a high predictor of self-efficacy in mathematical literacy, it also appears that the level of education is a factor. Lubinski and Benbow (2006) found that graduate students are more likely to identify STEM courses as their favorite class. They found these results using a longitudinal study over 35 years to determine whether ability levels and confidences cross-over to adulthood (Lubinski & Benbow, 2006). By their mid-thirties, men and women appeared to be happy with their life choices and viewed themselves as equally successful in the fields of mathematics and science (Lubinski & Benbow, 2006). They were thus proving that aging has a positive correlation to the self-efficacy of math and science students.

Teaching Mathematical Literacy

To increase mathematical literacy rates across the field of mathematics, current research is instrumental in determining what strategies will be useful. Treating mathematical literacy similarly to other content areas (reading literacy) may work sometimes but could be unreasonable and too simplistic thus glossing over the complexity of mathematics. It is unclear which elements definitively overlap. This section on teaching mathematical literacy strives to identify factors affecting educators to help them be successful in their practice.

Math anxiety is one limiting factor prevalent to today's youth (94% affected), and previous research supports the findings that it contributes substantially to lower achievement rates (Radisic et al., 2015). External factors can also contribute towards higher math anxiety. Research has discovered that instructors may be substandard at teaching students *how* to prepare for math assessments (Radisic et al., 2015). This impacts the results of determining what students actually understand regarding mathematics, either displaying the lack of assessment-based application practices or a misunderstanding of content taught. Whether it is inexperienced teaching, the unfamiliarity of the curriculum used, or rewriting assessments consistently (all legitimate considerations impacting assessment); students need more help in *how* to navigate and familiarize themselves with the process of test-taking from quality educators. Another external factor relates to teachers who inadvertently pass on anxious tendencies through their teaching demeanor to students, especially along gender lines (i.e., female teachers to female students) (Maloney & Beilock, 2012). Maloney and Beilock (2012) point to female educators' feelings of inferiority surrounding mathematics which is associated with being a male-dominated content area. Research indicates female students are more susceptible, possibly due to socialization, to recognizing and internalizing instructor expressions of math anxiety versus male students

(Maloney & Beilock, 2012). Both of these factors can be related to teacher preparedness programs, which need to address the pathways of learning mathematics and stereotyping effects on students and educators.

To ascertain how to change the teaching process to increase mathematical literacy, it is helpful to understand existing teacher preparation programs. Previously, research was not completed to determine what content-specific knowledge relates to student success, making it challenging to infer aspects of importance (Hill, Rowan, & Ball, 2005). An investigation completed by Hill et al. (2005) explored how a teacher's education and mathematical knowledge contributed to student achievement and teacher quality through preparatory courses and hands-on experience (Hill et al., 2005). This research included three variables in determining teacher quality: teacher certification, educational coursework, and experience. The sample population included teachers of 1,190 first-graders and 1,773 third graders, representing 42 districts in 15 states (Hill et al., 2005). They established that 12% of teachers had never completed a teaching mathematics course, 90% were certified educators, and the average teaching experience was 12 years (Hill et al., 2005). These findings suggest that just increasing the amount of required math content coursework versus best practices for teaching math is insufficient and both should be considered equally important if one is to become a math teacher. Hill et al. (2005) proposed the enrichment of teacher preparation programs' because content knowledge (math and *how* it's taught) was a significant predictor of student mathematical gains. With this information in mind, improving content knowledge to support mathematical literacy seems fundamental.

Through the improvement of teacher preparation programs, educator self-efficacy, that is one's confidence in their role as a math teacher, should therefore increase. Arslan and Yavuz (2012) set out to discover the prevalence of feelings of inadequacy when they interviewed 140 prospective mathematics and physics teachers. Unfortunately, the authors concluded that all participants identified as below average, but benefited from a self-evaluation and reflection process to express their anxieties before they are brought to their student's attention (Arslan & Yavuz, 2012). These prospective teachers were on alternative career paths when beginning their graduate studies and it is unclear whether this is a contributing factor (Arslan & Yavuz, 2012). In conclusion, the improvement of educator confidence and self-efficacy beliefs would be favorable for inexperienced teachers (and their students), but this article did not offer constructive suggestions for how (Arslan & Yavuz, 2012).

Bonner (2006), a professor at Azuza University in Southern California, completed an action research project to elicit teaching transformation through a mentorship program. It was through her work with two fifth-grade teachers that Bonner (2006) could give specific educators opportunities for reflection in a team-teaching arrangement. Her goal was to create positive attitudes and a sense of instructional expertise for an experienced educator (13 years of service) and a fourth-year teacher (Bonner, 2006). These teachers taught 30 to 32 low socioeconomic status students each, with over 50% speaking Spanish (Bonner, 2006). Through their partnership, both educators admitted to gaining a wealth of knowledge for teaching math and this was evidenced through new ways of thinking about learning, their students, and themselves (Bonner, 2006). Bonner (2006) hoped to align this project to Bertalanffy's General Systems Theory (1968) which states, "a change in one part of the group affects some of all the others" (as cited in Bonner, 2006, p. 28). It was while assisting these teachers that Bonner (2006) believed that changing the self-efficacy of a small portion of the population could start a chain reaction, affecting the overall system of education.

Many external factors contribute to increased levels of math anxiety and consequently decreased self-efficacy. By examining teacher preparedness and explaining the process of how to study and teach mathematics, researchers hope to assist administrators to improve the effectiveness of teachers. It is instrumental that educators receive formal training and support through the implementation of new mathematical literacy promotion strategies. The more prepared individual teachers are, the more likely they are to use these new methods.

Math Literacy Strategies

Teachers are vital in the development of strategies used to foster mathematical literacy because of their understanding of the students and the tools they incorporate into their daily lessons. In the traditional school setting, students are learning theoretical frameworks that can be considered abstract and do not transfer across subjects well (Wicklein & Schell, 1995). This prevents knowledge from being activated for further use beyond the intended application, and if knowledge has no apparent application, then it can be classified as unmeaningful (Wicklein & Schell, 1995). Mathematical concepts become isolated from a community of students and start to be considered pointless except as pertaining to a math problem or in an abstract way (Wicklein & Schell, 1995). Wicklein and Schell (1995) recommend that interdisciplinary lessons can be a more realistic and plausible solution because it shows how these subjects are intertwined. By placing these content areas in unison to achieve a task, educators are mimicking real-world scenarios.

Carter and Dean (2006), investigated the incorporation of specific reading strategies (vocabulary, comprehension, and decoding) into mathematics tasks by evaluating 72 lessons. Approximately 70% of the instances in this study were vocabulary instruction and likely attributed to teachers recognizing the importance of terminology in the mathematics classroom

(Carter & Dean, 2006). Students were given multiple strategies to determine those advantageous for overall mathematical comprehension. Results indicated that the use of prior knowledge, comprehension of vocabulary, and monitoring of reading were the most successful (Carter & Dean, 2006). Although the research found the least number of instances of decoding (sounding out words you do not know, Latin roots, contextual clues), compared to vocabulary and comprehension, that is not to say this is unimportant (Carter & Dean, 2006). The authors attribute the infrequencies to teachers being inadequate at teaching the strategy of decoding or scarcity of opportunities offered to practice this skill independently (Carter & Dean, 2006). Unfortunately, this study did not include suggestions for improvements but it did offer insight into the prevalence of literacy learning techniques in the mathematics classroom. The authors concluded with the statement that the reading teacher should not hold all the responsibility of increasing literacy but should be the responsibility of all content area teachers (Carter & Dean, 2006).

Adams and Pegg (2012) strived to understand some of the many variables that contribute to the enactment of literacy strategies (anticipation guides, picture dictionary, definition summarization) in math and science classrooms related to vocabulary and comprehension. The population included 26 science and math teachers chosen from a high needs district to participate in a qualitative inquiry involving observations and reflections (Adams & Pegg, 2012). Results found that the teachers tended to enact content literacy strategies in ways that aligned with instructional goals and current practices (Adams & Pegg, 2012). It is stressed that educators need to be careful during implementation because the way in which they teach it has the largest influence over students meeting the learning target of the strategy or comprehending math vocabulary (Adams & Pegg, 2012). Professional development opportunities or teacher preparation programs should be offered to support educators by providing evidence of the ways in which educators have used these strategies (i.e. anticipation guides, picture dictionary, definition summarization) to deepen student understanding (Adams & Pegg, 2012). By increasing vocabulary and comprehension in math, literacy improves in all other subjects and vice versa.

Borasi, Siegel, Fonzi, and Smith (1998) introduced instructional experiences for educators as a simulation of transactional reading strategies (paraphrasing, reading and reflection, and creating visual representations) in connection with a variety of mathematics-related texts. Due to the call for connecting mathematics to the real-world through reasoning, problem-solving, and communication these strategies encourage readers to revise their interpretations, consider alternative perspectives, and even generate and pursue new questions that might go beyond the content of the text itself (Borasi et al., 1998). Instructional practices drawn from the field of reading can contribute more to mathematics instruction through a reciprocal approach. The findings suggest that a sense of community is facilitated when negotiating meaning (revising interpretations, considering alternative perspectives and generating new questions) in mathematics through the exploration of transactional reading strategies (Borasi et al., 1998). By modeling this community importance, educators can help students to create a supportive environment where students feel sale to share their ideas.

Another strategy by Leibowitz (2016) examined methods used by one introductory algebra teacher to support ninth-grade students by individually examining their understanding of the distributive property in algebra. The participants of this study included one teacher from an urban high school utilizing visual cues among her African American students consistent with the teacher’s use of dialogue, gesture, and references (Leibowitz, 2016). In addition, the use of arrows and illustrations was present extending current research in determining how to develop mathematical literacy skills using verbal and visual cues (Leibowitz, 2016). Other recent research has indicated that all content areas differ in purpose, symbols, communication, and language, suggesting there is no one way to teach literacy across disciplines (Leibowitz, 2016). It is acknowledged that the importance of language and literacy in mathematics is a new development recently arising in the creation of the Educative Teacher Performance Assessment (edTPA) in 2014 (Leibowitz, 2016). Not every preparation program is required to utilize this assessment, which may result in inconsistent literacy development amongst educators and therefore students.

Remembering that math literacy encompasses a student's ability to make mathematical connections with the real-world, Kapur (2014) suggested that through the strategy of productive failure; differentiation and prior knowledge activation gave students more attention to detail. The process of productive failure provides unique opportunities to have students learn from their mistakes. The study included ninth-grade students in two separate instances in an Indian private school. It was found that students used more mental effort but significantly outperformed other students on conceptual understanding and the transferring of procedural knowledge when using productive failure (Kapur, 2014). The benefit of productive failure is it allows students to learn procedures piece by piece, thereby allowing for an opportunity to learn on their own which can be helpful in mathematical literacy (Kapur, 2014). Though a sense of independence in the classroom is beneficial for learning and self-efficacy, educators should be presenting productive failure group tasks to ensure students receive immediate and accurate feedback on new content.

Activating prior knowledge has been identified by Heidema and Jordan (2002) as being the pivotal skill when learning a new mathematical concept. By bringing previously learned ideas, students will better learn and remember the new vocabulary and text (Heidema & Jordan, 2002). Preparing lessons for teaching vocabulary effectively identifies opportunities for students to connect what they have learned during math class or in other subjects to solidify understanding (Heidema & Jordan, 2002). Utilizing anticipation guides activates prior knowledge in preparation for analyzing new material by asking students to respond to opinions challenging what they believe to be true (Heidema & Jordan, 2002). This activation of knowledge can be helpful in the productive failure strategy as mentioned by Kapur (2012) or independently in the beginning stages of mathematical literacy.

Many students come prepared for math class with basic reading skills that scarcely prepare them for reading mathematics texts, where a specialized set of skills is required. For example, students are forced to comprehend the text, decipher symbols, and decode new vocabulary; in every direction (Heidema & Jordan, 2002). These works contain larger and longer content and the style of the text can greatly impede reading comprehension (Heidema & Jordan, 2002). Authors of mathematics and scientific publications are able to stray from the typical principles of writing found in language arts (i.e. the main idea may not appear at the beginning) (Heidema & Jordan, 2002). By utilizing these varying formats, students must infer and use their prior knowledge to find relationships amongst the text (Heidema & Jordan, 2002). Heidema and Jordan (2002) suggest that students be taught to survey, question, read, question, compute, and question (SQRQCQ). This strategy is best utilized for tackling hard to read word problems that many students struggle to decipher and practically apply math concepts (Heidema & Jordan, 2002). By lacking an understanding of the process to approach these problems, it emphasizes the importance of mathematical literacy and teacher collaboration.

In conclusion, it is with these strategies that educators strive to promote mathematical literacy in the classroom. Research completed on this topic includes (but is not limited to): productive failure, verbal and visual cues, transactional reading strategies, vocabulary, decoding, comprehension, interdisciplinary teaching, activating prior knowledge and navigating math texts. Creating learning tasks that take into account the needs of the student to develop a real-world application utilizing prior knowledge can greatly improve literacy tendencies in the mathematics classroom.

Tangible Teaching

One such approach is to present symbolic representations using concrete objects. This style of teaching is termed *tangible teaching* and is in response to a call for active learning. Scarlatos (2006) set out to increase the prevalence of tangible teaching by creating online programs to simulate popular math manipulatives such as pattern blocks and number tiles. The research found that these programs help students to remain focused, think in new ways, and develop problem-solving skills to increase critical thinking (Scarlatos, 2006). Parameters to incorporate when creating these mathematical programs (i.e. Geogebra and Desmos) include physical activity, collaborative learning, elements of fun, immediate feedback, information gathering, and teacher customization (Scarlatos, 2006). Each of these facets helps to address the multiple intelligences found in the classroom and allows students to scaffold off of their peer's prior knowledge (Scarlatos, 2006).

Carbonneau, Marley, and Selig (2013) analyzed 55 studies based on the effect of using concrete manipulatives. They compared instructional techniques that used manipulatives with a comparison group that taught math with only abstract math symbols. They also used a form of instruction where students were able to learn from manipulatives and enough quantitative information to estimate an effect (studies that required students to work with rulers, scales, or calculators were not included). Carbonneau et al. (2013) found that using manipulatives in mathematics instruction produced a small- to moderate-sized effect on student learning when compared with instruction that used abstract symbols alone.

Results also revealed that the strength of this effect is dependent upon other instructional variables (instructional guidance, type of manipulative, and instructional time). Just incorporating manipulatives into mathematics instruction may not be enough to increase student achievement or math literacy (Carbonneau et al., 2013). For example, concrete manipulatives are the most beneficial when learning new content, and in order for students to gain an understanding of mathematical literacy, they must participate and engage with these items over an extended period of time (Carbonneau et al., 2013). Further studies, which take into account the types of manipulatives, are being undertaken, but it is suggested that the more representations made available, the more successful a student will be (Carbonneau et al., 2013).

Research on modeling, gesturing, and quantitative literacy completed by Wilkins (2010) as well as Goldin-Meadow, Cook, and Mitchell (2009) offer suggestions on tangible ways of teaching. Wilkins' (2010) purpose was to relate the components of quantitative literacy from a psychological perspective and investigate the model that encompassed these relationships. He based his framework on a quantitatively literate person possessing the following: mathematical reasoning skills, real-life applications of mathematics, a historical overview, a positive attitude towards and knowledge of mathematical content (Wilkins, 2010). Wilkins (2010) hypothesized that the general construct of quantitative literacy was a hierarchical three-factor model that took into account beliefs about mathematics, mathematical cognition, and disposition towards mathematics. His findings were in support of this model and indicated that using these considerations when developing a tangible mathematics curriculum is important to determine an effective assessment tool for quantitative literacy and the acknowledgment of math literacy as a multifaceted construct (Wilkins, 2010). To date, this has not existed though attempts have been made and future research should focus on the identification of assessment measures to increase reliability (Wilkins, 2010).

Goldin-Meadow et. al. (2009) recognized the impact of relevant curriculum when using manipulatives and strived to answer the question: how does gesturing help students learn? This study consisted of 128 students (aged 9-10) who completed a pre and post-test containing addition problems and the data suggests that gesturing can facilitate learning by helping children extract information from their hand movements. Body movements are a part of how people learn--they involve processing old ideas, but also create new ones (Goldin-Meadow et al., 2009). Using these tangible techniques in teaching and learning, students with learning disorders or decreased attention spans can feel successful and the authors wondered if the gesturing helped to focus their attention on the task at hand (Goldin-Meadow et al., 2009, p. 270). Utilizing these physical components in mathematics provides the educator with an opportunity to link abstract concepts to real-world scenarios.

Tangible teaching allows students to activate prior knowledge, create a learning community, and make connections to practical events. By adopting and facilitating this array of literacy teaching strategies, educators are providing students with various types of mathematical representations, therefore appealing to their multiple intelligences. Aligning this with student-centered learning by providing students a toolbox of methods to decipher, decode, and comprehend new math promotes positive self-efficacy and increases retention. Teachers and families can rally around the learner to create a web of support and as a result, should be able to identify inhibiting factors of mathematical literacy immediately upon their presentation, preventing students from falling between the cracks in the educational system.

CHAPTER III: DISCUSSION AND SUMMARY

Summary of Literature

The United States Constitution relied on three forms of quantification (census, taxes, and ⅗ compromise) utilizing the skills of political arithmetic, which originated in Colonial America (Cohen, 2003; Steen, 2001). Upon entering the 21st century, the country required a comprehensive understanding of mathematics, labeled *mathematization*, consisting of making connections between abstract and practical (Rizki & Priatna, 2019). The NCTM (2000) and OECD (2013) responded to Beaudine's (2018) 20-year request for implementing reading into the mathematics classroom by identifying necessary components of prose, document, and quantitative literacy. These strategies created a shift in education promoting well-formed productive citizens fluent in the language of mathematics (Sasanguie et al., 2012). Unfortunately, this delay resulted in the United States ranking lower than 36 other countries on mathematical literacy, leaving corporations concerned with their employees' skills (PISA, 2018; Rhodes, 2010).

Mathematical literacy and proficiency were used as synonymous terms by several researchers (Kilpatrick, 2001; Boyer & Merzbach, 1991; Ippolito et al., 2017). Kilpatrick (2001) stated that "mathematical proficiency can be used to define learning goals for all students" through its facets of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (p. 106). By utilizing proficiency versus literacy, Kilpatrick (2001) hoped to "avoid the extreme positions with respect to the goals of math learning" (p.106). Wilkins (2000) concluded that to ensure students are mathematically literate, they must possess "a functional knowledge of mathematical content, an ability to reason, a recognition of the societal impact, an understanding of the nature and history of mathematics, and a positive disposition towards the subject" (p. 406). Conversely, Yore, Pimm, & Tuan (2007) indicated there is a significant difference between proficiency and literacy stating, "literacies have less to do with school curricula and more to do with learning and the intentions and effects of schooling on democratic citizenship and adult life" (p. 559). This proves there is a potential convergence between proficiency and literacy in the specification of mathematical literacy.

Rizki and Priatna (2019) indicated that the reasoning for choosing one term over the other may be that "mathematical literacy is still foreign to some societies, yet it is important for the society in the 21st century" (p. 1). The most commonly used definition in the current literature is provided by the OECD, this organization's findings show that the United States is in a STEM race with the rest of the developing world, falling behind 36 other educational systems (Haara et al., 2017; OECD, 2013; PISA, 2018). Likely motivated by these findings, researchers were determined to make changes in the field of mathematics education.

To improve mathematical literacy, factors that impact students must first be identified. Examples include learning disorders, ELL, math anxiety, age- and gender-related factors, stereotypes, and self-efficacy. Math learning disorders are a limiter of mathematics achievement affecting approximately 10% of the student body in the United States (Barbaresi et al., 2005). The data of Barbaresi et al.'s (2005) study suggests "Math LD is common among schoolchildren" resulting in "mathematical ability that is substantially below that expected for chronological age, intellectual level, and educational experience" (p. 281). Identifying factors that impede students allows educators to anticipate strategies and practices to utilize within the classroom.

While services are available for the previously stated learning barriers, English language learners represent a large portion of the population at risk of being mathematically illiterate.

Cramer (2013) proposed that mastering a new language is a complex process that includes basic interpersonal relationship skills (two years) and cognitive academic language proficiency (5 years). The research completed in this area found that “English proficiency precedes mathematical proficiency” proving the importance of accommodating ELL students (Henry et al., 2016, p. 24). Moschkovich (2015) recommends that to support these students, “opportunities for ELs to focus on language should be connected to the mathematical activity, instead of giving students definitions in a vacuum or divorcing language work from analytical work” (p. 57). Students who need language assistance should be offered the same support as those with intellectual learning factors.

Learning barriers effectively inhibit students more than their biological traits, as mental health diagnoses rise. As of 2015, 94% of youth are diagnosed with math anxiety, an increasingly problematic form of anxiety classified by avoidance behaviors caused by hyperactivity in the amygdala (Maloney & Beilock, 2012; Punaro & Reeves, 2012; Radisic et al., 2015). High levels of math anxiety and low socioeconomic status are directly correlated to lower achievement (Ankita & Richa, 2017; Radisic et al., 2015). These instances of math anxiety were thought to arise in the early high school years, but it has since been identified that they are occurring at much younger ages (Punaro & Reeves, 2012).

More general age-related differences (following Piaget’s developmental stages) attribute to a significant level of mathematical literacy (McLeod, 2018). Cleary and Chen (2009) found that due to developmental challenges (particularly educational transitions, i.e. elementary to middle school), seventh-grade students were less interested in mathematical activities. Additionally, middle school students find the most difficulty in algebraic concepts and use of variables (Setiawati et al., 2017). As children age, their mathematical precision increases, and their numerical reasoning decreases (Sasanguie et al., 2012). Age-related factors distinguished by developmental changes can impact the advancement of mathematical literacy.

Gender-related differences are not as easily documented, and it is unknown how they relate to stereotypes. Gender-related research completed by Lailiyah (2017) found that male subjects were more detail-oriented than female subjects. On the other hand, a longitudinal study by Lubinski and Benbow (2006) found that females exhibited higher-levels of interest and verbal-linguistic abilities ("tend to co-occur with social, or organic, interests and values"), making them a more substantial portion of the high-level workforce (p. 337). When discussing gender-related differences, it is statistically challenging to determine a large amount of disparity between males and females, but some studies have found that students are affected by gender stereotypes, even when proven to lack merit (Lindberg, Hyde, Petersen, & Linn, 2010). Stereotypes prey on a student's self-efficacy which in turn limits an individual's confidence in their learning capacity; examples include feeling prepared on an exam or thinking math is "easy" (Bandura, 1977). To determine what can influence student self-efficacy in a math classroom, researchers looked at biological, learning, and attitude factors. For example, ninth-grade students presented with high levels of math confidence, and the higher the math course the student takes, the more confident they become (Clearly & Chen, 2009; Ozgen & Bindak, 2011). This could be linked to the finding that students with a higher ability level are more likely to appreciate the content and view themselves as successful learners (Lubinski & Benbow, 2006). Bryan and Bryan (1991) found that students/educators who experienced happier thoughts before completing an assessment showed a significantly higher level of success. Typical complaints heard in mathematics' classrooms include my "teachers did not explain how the mathematics topics could be used in the real world," making it challenging for students to recognize real-world applications of the content (Ozgen, 2013, p. 313). Wicklein & Schell (1995) concur that students are learning abstract concepts that they find impossible to find meaning in.

Educator self-efficacy is confidence that a teacher has when teaching a content area to a group of students. Arslan and Yavuz (2012) determined through their research that all prospective teachers do not experience high levels of self-efficacy, though it is unclear why. The greater content specific courses a teacher completed in their preparation program, the more assurance they presented (Gulten, 2013). Hill et al. (2005) discovered that 12% of the teachers they studied had never completed a mathematics teaching course. Ninety percent of the previously identified group were certified educators, with an average of 12 years of math teaching experience (Hill et al., 2005). Researchers decided that teachers do not know what to prioritize in the classroom (Haara et al., 2017). In a similar situation, Bonner (2006) was able to give one-on-one training and reflection to two educators through mentoring and it was due to this experience she recognized that individual training opportunities like this could affect the system as a whole.

To assist teachers through the promotion of mathematical literacy in the classroom, researchers recommend utilizing instances of decoding within a lesson, and it is most useful to implement these strategies in alignment with instructional goals (Adams & Pegg, 2012; Carter & Dean, 2006). The findings of similar studies suggest that "when the teacher considered the exploration and negotiation of meaning as a valued way to learn mathematics, the use of transactional reading strategies provided the class with ways of working together that contributed to the development of a community of practice" (Borasi et al., 1998, p. 303).

The overarching goal of this research was to determine best practices related to increasing mathematical literacy. Carbonneau et al. 's (2013) research indicated that using manipulatives had measurable to small effects on student learning when compared with symbolic teaching. Data also suggests that gesturing can facilitate learning through hand movements (Leibowitz, 2016). However, it is hypothesized that the distraction of the movement increases the focus on the designated task (Goldin-Meadow et al., 2009). Scarlato (2006) offered the possibility of utilizing online tangible teaching methods to make more resources available. The most statistically significant data is in response to math students learning from their own mistakes. This characteristic is the most potent variable on mathematical literacy (Kapur, 2014). Wilkins (2010) developed a hierarchical model to assess the effectiveness of tangible teaching methods on acquiring mathematical literacy skills.

Samuelsson (2008) found that no single method affects all areas of mathematical proficiency with the same impact. Through the use of a variety of strategies, educators can appeal to the multiple learning styles that students possess. Teachers need to receive proper training and preparation to adequately teach these practices.

Professional Applications

As an educator in the field of mathematics, it is essential to stay current on teaching strategies. This research supports choosing an appropriate curriculum that includes real-life components to teach children practical math, therefore improving mathematical literacy. As a nation, the United States is seeing a decline in proficiency levels of mathematics in comparison to other world powers. It would be realistic to state that the current educational system is faulted, but increasing a student's ability to identify and utilize real-world concepts should always be at the forefront of a mathematics lesson. Therefore, there is a need to establish the difference between teaching for the assessments versus creating constructive citizens with critical thinking abilities.

Teacher preparation programs should be helping educators teach mathematics for the 21st century. Prospective educators need to be shown *how* to teach math to increase literacy levels and be given strategies to facilitate these changes. The research shows that high-level mathematics courses do not equate to more adept teachers of mathematics and therefore proposes the completion of mathematics coursework focused on teaching methods.

The current educational system has many different schedules. One common example is the trimester schedule which creates challenges for students to take math all year because they are typically taught utilizing two or the three trimester periods. Including summer break, students can have a span of nine months to forget math content, consequently decreasing proficiency levels. Additionally, most high schools only require three years of math, thus setting college freshman up for failure in introductory mathematics courses.

Adjacent to systemic changes, educators should promote mathematical literacy by offering students an opportunity to identify mistakes and correct them. The literature classifies this exercise of productive failure as *the* critical device of all mathematical learners. To utilize this methodology, the research recommends that students should be able to make test corrections and have one-on-one appointments with a teacher to go over a recent examination. This would allow students to increase repetitions of each problem or type of problem.

Limitations of the Research

To limit current research, studies completed in the United States were made the primary focus. It was necessary to include research completed within the last thirty years to span numerous educational reforms. There is an abundance of keywords and catchphrases associated with mathematical literacy, and therefore it was essential to narrow the search. The keywords used in these searches included: "content literacy strategies", "math", "math reading comprehension", "professional development math literacy", "professional development content literacy", "math literacy", and "quantitative literacy". This collection of topics provided an abundance of articles, but because the definition of mathematical literacy or proficiency is different depending on the author, it was challenging to find consistent information.

In the world of education, teachers are always striving to find out the best way to reach their students. Attempting to discover the secret of mathematical literacy seemed to be a natural progression. The research was unable to provide a statistically significant universal phenomenon that can be replicable within every classroom. There were small contributions, such as socioeconomic status, teacher responsiveness, and the method of exposure to new material.

Finally, most of these articles referenced the use of the PISA mathematical literacy study protocol and criteria. The National Council for Teachers of Mathematics (NCTM) should therefore acknowledge this as the standard for assessing mathematical literacy, thus, universalizing the screening system for relevant research.

The current research lacked extensive examinations of diverse populations. This posed challenging when trying to determine the validity and reliability of the literature because these findings may not be indicative of every learner or type of learner. Additionally, the investigations are vague when identifying familial roles in mathematical importance, attitude, and self-efficacy. The research has addressed the intensity of the impact, but not the interventions that have proven to be useful. Due to the strong connections between family members and the amount of diversity in the United States, the predictive impact of these factors needs further analyzation.

Implications for Future Research

The findings of mathematical literacy research need to be replicable for it to be utilized in the classroom or throughout education. Further research may need to focus on large scale studies that include variables such as employing a broad population to dive into gender, grade-level, rigor, and age-appropriate factors. Without the use of a large sample size, the diversity found among participants may not be due to specific characteristics but rather from slight variations in the methods.

Significant research should also be completed on the professional development opportunities that can be directed towards mathematics teachers of all ages and how one can maximize an educator's potential. To help reduce the responsibilities of the teacher, it would be beneficial to review the current curriculum to determine its effectiveness in mathematical literacy. Not only should the focus be on the math learning of a student, but also facilitating traits that increase a student's mathematical success. It would be useful to research how these traits occur in the math classroom and compare it with the benefits of these characteristics in other parts of life (i.e. in the workforce).

The current research raises the question of how English language learners are taught. If the research states that English proficiency precedes mathematical proficiency, educators must be adopting that timeline. More studies should include English language learners and analyze factors that inhibit them in the field of mathematics. Some of these factors may also be affecting the entire student body.

Conclusion

In response to the research question, "what are common traits of tenacious mathematics students, and how can these commonalities be used to aid the teaching community in the promotion of mathematical literacy?", this literature review found that extrinsic variables, rather than intrinsic, primarily impact students. Therefore, the way that mathematics is taught throughout the educational system as a whole is a more significant factor in a student's success than their backgrounds or characteristics. To overcome shortcomings, the United States must lead by example through the promotion of impactful math experiences in hopes of shifting the paradigm in this mathematically literate era.

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