
Alisha A. Joyce
Bethel University

Follow this and additional works at: https://spark.bethel.edu/etd

Part of the Elementary Education Commons

Recommended Citation

This Master’s thesis is brought to you for free and open access by Spark. It has been accepted for inclusion in All Electronic Theses and Dissertations by an authorized administrator of Spark.
TEACHING FOR LEARNING: CREATIVE METHODS FOR MATH CURRICULUM DESIGN
USING COGNITIVE SCIENCE PRINCIPLES

A MASTER’S THESIS
SUBMITTED TO THE FACULTY
OF BETHEL UNIVERSITY

BY
ALISHA JOYCE

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF ARTS IN EDUCATION

July 19, 2020
TEACHING FOR LEARNING: CREATIVE METHODS FOR MATH CURRICULUM DESIGN
USING COGNITIVE SCIENCE PRINCIPLES

Alisha Joyce

August 2020

APPROVED

Thesis Advisor: Nathan Elliott, M.A.

Program Director: Lisa M. Silmser, Ed. D.
Acknowledgements

Thank you to Andrew Joyce and Karen Hange, who impacted this project with their support, encouragement, and time spent listening. Thank you to my advisor, Nathan Elliott. Your flexibility, patience, advice, and guidance have brought me through!

And a final thanks to my students: Adiya, Da’Mylah, Celeste, Elizabeth, Emorionne, Hudson, Janice, Jordin, Makaela, Malachi, Melody, Miciah, Oliver, Savaeh, Sophia, Vicente, and Zaden. You are my inspiration, my motivation, and my adoration.
Abstract

It has long been the goal of educators to teach for lifelong learning. This project is an examination of best practices for teaching for lifelong learning in the context of an elementary math curriculum. In pursuit of this research, I identified two guiding questions: What are the best practices for teaching and assessing mathematics in elementary school? How can these ideas be organized into an adaptable format for use in planning instructional units? In the research, there is a general consensus that educators need to shift their instruction to a goal of teaching students a conceptual understanding of mathematics, making connections between processes and skills. Because of this, it is effective to use practice that emphasize connections between skills, such as project-based learning and concept mapping. The final product for this application project synthesizes the research to create a unit plan that incorporates elements of project-based learning, standards-based grading, formative assessment, brain-based learning, and use of visual strategies and manipulatives.
# Table of Contents

 Signature Page .................................................................................................................. 2  
 Acknowledgements .......................................................................................................... 3  
 Abstract ............................................................................................................................. 4  
 Table of Contents ............................................................................................................. 5  
 Chapter I: Introduction ..................................................................................................... 7  
    Theoretical frameworks .................................................................................................. 7  
    Instructional strategies .................................................................................................. 9  
    Guiding questions ......................................................................................................... 11  
 Chapter II: Literature Review .......................................................................................... 13  
    Literature Search Procedures ...................................................................................... 13  
    Integration of Teaching, Learning, and Assessment ..................................................... 13  
    Cognitive Processes ...................................................................................................... 14  
        Foundational Processes ........................................................................................... 14  
        Memory Processes .................................................................................................. 15  
        Visual Processes ..................................................................................................... 17  
        Socio-Emotional Processes ..................................................................................... 19  
    Instructional Strategies .................................................................................................. 20  
        Strategies for Foundational Processes ..................................................................... 20  
        Strategies for Memory Processes ............................................................................. 22  
        Strategies for Visual Processes ................................................................................ 24  
        Strategies for Socio-Emotional Processes ............................................................... 27
Conclusion

Chapter III: Research Application

Chapter IV: Discussion and Conclusion

Summary of Literature

Limitations of the Research

Implications for Future Research

Implications for Professional Application

Conclusion

References

Appendix
CHAPTER I: INTRODUCTION

The world is often arranged into a dichotomy: the arts and the sciences. Dichotomies can be useful ways to sort things, but the problem is that there is always something which will be made incomplete by its placement in a single category. Teaching is one of those things. When the teacher is only an artist, learning lacks utility. When the teacher is only a scientist, learning lacks permanence. I am an elementary teacher, and I am also an artist. (I have never been a scientist.) This research is an artist’s foray into the science of learning.

Theoretical Frameworks

The inclination to let learning set the stage for teaching is not new. Cognitive science has been a catalyst for the development of many educational trends throughout the years. In 1997, Joyce Wolfer Bishop sought to answer the questions about what strategies students were using to solve problems, and how these strategies related to the “symbolic representations” they developed. She concluded that students develop algebraic knowledge through an awareness of the “underlying relationships” between problems (Bishop 1997). Later, Chesney (2014), argued that traditional methods of operational thinking were ineffective, and that teaching relational ways of thinking has a better long-term impact on learning.

Cognitive science research continued, and in 2006, two educators collaborated to consolidate this research into a collection of key principles that teachers could use to guide their instructional planning. Among these principles is the idea that understanding cannot be given, but rather must be engineered by the teacher and experienced by the
student (Wiggins & McTighe, 2006). Research about the process of learning has led some educators to develop a system of foundational elements, or building blocks of learning (Washburn, 2010). Wilkerson and Wilensky (2011) conducted a study to analyze how expert mathematicians followed a similar process to understand new mathematics to which they had not been previously exposed, thus outlining a plan for the future engineering of instructional content.

Specific elements of cognitive research, such as conceptual blending, visuo-spatial skills, executive function, working memory training, and retrieval practice, have been the focus of many studies in recent years. Zandieh (2011) illustrated the effectiveness of using conceptual blending as an instructional tool for learning mathematics. Two separate studies by Honore and Noel (2017) and Zhang (2017) examined the effectiveness of working memory training on different mathematical skills. Other research has confirmed that explicit teaching of executive function visuo-spatial skills are critical for students’ mathematical achievement (Kim, 2016; Jamil & Ghazali, 2018). Most recently, Agarwal (2019) found that retrieval practice was most effective when both factual knowledge and high-order thinking skills were assessed. This further confirms earlier beliefs that conceptual knowledge is more important than operational knowledge (Bishop 1997; Chesney & McNeil 2014).

Other researchers focused on specific aspects of students’ internal process to examine the impact on learning. Immordino-Yang and Damasio (2007) argued that high-level cognitive skills are directly related to socio-emotional development. Powell and Fuchs (2014) are two researchers who have conducted many studies on the effects of
performance based instruction on student’s attitudes and achievements, and their research was confirmed by Arhin (2015) who liked positive correlations in both attitude and achievement through performance based assessment.

**Instructional Strategies**

In all of this research, a key theme has been moving students beyond fact recall and into higher order thinking and application of mathematic concepts and skills. Researchers Black and Wiliam framed the problem in this way: “The tests used by teachers encourage rote and superficial learning even when teachers say they want to develop understanding [...] The giving of marks and the grading function are overemphasized, while the giving of useful advice and the learning function are underemphasized” (Black & Wiliam 1998, p. 83). Many educational trends have surfaced as teachers sought to put these theories and findings into practice in their own classrooms.

An early educational strategy was the use of performance assessments to drive learning. In 1995, a team of researchers observed that after using performance assessment as an instructional method, teachers were more likely to use hands-on activities and problem-solving strategies in their classes, which they saw as a positive impact on both teaching and learning (Flexer, Cumbo, Borko, Mayfield, & Marion, 1995). The validity and reliability of performance assessments was confirmed the next year by another team of researchers (Brown, O’Gorman, & Du 1996). Other educators echoed Black and Wiliam’s research in a compilation of practical teaching strategies for using
formative assessment and instructive feedback (Leahy, Lyon, Thompson, & Wiliam 2005).

The emphasis on performance assessment led many educators to design instructional frameworks that focused on problem solving and authentic experiences to drive learning in mathematics (Althausser, 2016; Magee & Flessner, 2012; McGregor, 2014). Recently, a team of educators developed the Design Thinking Framework, which serves as a model for other teachers to use in developing these authentic experiences (Bush et al., 2018). These kinds of experiences activate the socio-emotional component of learning described by Immordino-Yang and Damasio (2007). In addition, other researchers have studied the effects of cooperative learning strategies on student achievement in mathematics (Thomas & Feng 2014).

More recently, standards-based grading has emerged as a trend to move teachers away from the focus on the grading function and towards the teaching function (Black & Wiliam, 1998). In fact, some researchers conducted an extensive study wherein they determined that standards-based grading had a higher correlation to student achievement than traditional grading systems (Lehman, De Jong, & Baron 2018).

Beyond assessment, other educators have focused on how to refine instructional strategies to follow a better process of learning. The use of schema based instruction and concept mapping have proven to be effective ways of teaching students mathematics, which confirms earlier research about the relationships between problems (Bot & Eze, 2016; Griffin, Gagnon, Jossi, Ulrich, & Myers, 2018; Powell & Fuchs, 2018).
In examining relationships between problems, some educators have developed strategies of using visual tools and graphic organizers in mathematics instruction (Delisio, Bukaty, & Taylor, 2018; Xin, 2018). Laski, Jor’dan, Daoust and Murray (2015), describe how manipulatives can be effectively used to move students from concrete, to pictorial, to abstract representations of problems. Kaur’s Model Method also makes use of this concrete-pictorial-abstract approach (Kaur, 2018).

Guiding Questions

Recent developments in educational research have led to increased implementation of practices such as inquiry-based learning, performance assessment, standards-based grading, cognitive science, schema-based instruction, interdisciplinary curriculum, and formative assessment. These trends are based in research of cognitive science, and many educators are now looking to the way children learn as a foundation for the way teachers teach. As an elementary teacher, I was looking for a way to synthesize many philosophies and instructional strategies into an effective plan for learning that could be used in my classroom.

In pursuit of this research, I identified two guiding questions: What are the best practices for teaching and assessing mathematics in elementary school? How can these ideas be organized into an adaptable format for use in planning instructional units? I began by examining recent studies related to best practices for teaching math, searching specifically for some of the aforementioned educational trends. Then, I determined connections between the philosophies for these practices. Finally, I
synthesized the research to create sample unit plans for use in my elementary classroom.

As a teacher, it can be overwhelming to keep up with educational trends and decide what methods of instruction will be most beneficial for students. These sample unit plans will be the foundation from which I plan to build my math units moving forward, and I hope that other teachers can benefit from these tools as well.
CHAPTER II: LITERATURE REVIEW

Literature Search Procedures

Research for this thesis was focused on discovering best practices for teaching and assessing mathematics. Recent developments in educational research have led to increased implementation of inquiry-based learning, performance assessment, standards-based grading, cognitive science, schema-based instruction, interdisciplinary curriculum, and formative assessment. These were topics that were pursued in examining articles from ERIC and Educator’s Reference Complete. Searches in these databases also included terms such as “conceptual blending,” “patterns in mathematics,” “concept mapping,” and “visual mathematics.” The primarily peer-reviewed articles were selected based on their relevance to recent educational trends in order to establish an array of practices to use in creating instructional materials for teachers that assist them in implementing best practices for teaching and assessing mathematics.

Integration of Teaching, Learning and Assessment

An examination of teaching must begin with an examination of learning. Teaching, learning, and assessment are three components of education that form a symbiotic relationship (Arhin, 2015; Black & Wiliam, 1998; Flexer, Cumbo, Borko, Mayfield, & Marion, 1995; Leahy, Lyon, Thompson, & Wiliam, 2005; Washburn, 2010). Each process informs the other. Learning informs teaching practices, as teachers may modify classroom activities and assessments to meet the needs of learners (Black & Wiliam, 1998). Assessment drives learning, as teachers use predetermined standards to
move students forward to a goal (Washburn, 2010). Finally, teaching has a direct impact on learning, which is then demonstrated by assessment (Leahy, Lyon, Thompson, & Wiliam, 2005). These processes are interdependent. Therefore, any attempt at curriculum design must begin with an examination of the work of the Designer.

**Cognitive Processes**

Learning process lays the foundation for teaching practice. The way that children learn informs both instruction and classroom practice (Flexer, Cumbo, Borko, Hilda, Mayfield, & Marion, 1995). Teaching that is aligned with cognitive science principles will be more effective, even when this means that “traditional” methods must be left behind (Arhin, 2015). In the following literature review, cognitive science principles are consolidated into four types of processes, which will then be used to connect to specific instructional strategies.

**Foundational Processes**

Learners use a variety of resources, or processes, for learning (Washburn 2010; Wilkerson-Jerde & Wilensky, 2011). Washburn identified five “building blocks” of learning: experience, which uses new data to establish a reference point; comprehension, wherein the brain breaks down new information; elaboration, which engages in connecting new ideas with known information; application, where the brain practices using new knowledge; and intention, where the new skills are applied to other contexts (Washburn, 2010, 8). Wilkerson-Jerde and Wilensky echoed these concepts in their study that sought to discover how accomplished mathematicians developed mathematical understanding. For this qualitative study, the researchers selected ten
advanced mathematicians from universities and PhD programs identified through faculty recommendations. The researchers gave the mathematicians an unfamiliar proof and asked them to think aloud in an interview process as they solved the proof. Their ideas were recorded according to a code, which led to the researchers identifying a series of resources that the mathematicians used to understand and solve problems in new concept areas. The mathematicians utilized prior knowledge (experience), broke down the new idea into distinct concepts (comprehension), connected the ideas to concrete examples (elaboration), developed new representations of the idea (application), and created specific constructions of the new idea to use in solving problems (intention) (Washburn, 2010, p. 8; Wilkerson-Jerde & Willensky, 2011, p. 29-30).

Research also supports the claim that in order to understand new information, learners must participate in conceptual blending (Wilkerson-Jerde & Wilensky, 2011; Zandieh, Roh, & Knapp, 2011). Conceptual blending is a process by which two concepts are interleaved to establish a new idea (Zandieh et. al., 2011). This cognitive process relates to the process of combining new information with known information.

Memory Processes

Many studies have supported the use of explicit instruction and training in retrieval practice, executive functions and visual working memory skills (Agarwal 2019; Honore & Noel, 2017; Jamil & Ghazali, 2018; Kim & Cameron, 2016; Zhang, 2017;). Retrieval practice is a learning strategy that involves engaging students in repeated recall of knowledge in spaced out sessions, so that the recall requires effort (Agarwal
Visual working memory is what enables the learner to create images and manipulate them mentally, or to visually represent concepts (Jamil & Ghazali, 2018). Executive function refers to the ability to control and direct attention to solving problems: ignoring distractions, planning and utilizing strategies, and storing information (Jamil & Ghazali, 2018).

Retrieval practice has been shown to increase learning when the students are engaged in repeated, effortful recall of knowledge (Brown, Roediger, & McDaniel, 2014). In a study on the effectiveness of retrieval practice, Agarwal (2019) conducted a series of three experiments. Experiments 1 and 2 were performed on 48 college students and evaluated whether higher-order or fact-knowledge question types used during retrieval practice was more effective in increasing test results. The evidence suggested that higher-order questions in retrieval practice were more likely to result in higher test scores. In Experiment 3, Agarwal attempted to find whether these same results would transfer to an authentic elementary classroom setting. After conducting a similar experiment on 142 sixth-grade students, the results were the same. Ultimately, Agarwal found that retrieval practice is most effective when the knowledge to be retrieved engages higher-order thinking, rather than low-level factual knowledge in relation to Bloom’s Taxonomy (Agarwal, 2019).

Zhang (2017) hypothesized that increased visual working memory training would have a direct impact on problem-solving performance for students with geometry difficulties. He designed a quantitative study that examined the effects of working memory training on four students who scored under the 30th percentile rank in both the
KeyMath geometry subtest and the SAT math test. The students were subjected to two intervention programs, one that focused exclusively on working memory training and one that was a combination of working memory training and direct instruction. Zhang found that working memory training alone was ineffective as a change agent. He advocates for the teaching of content knowledge and logical reasoning in tandem with visual working memory training to design an effective math curriculum.

Kim and Cameron (2016) conducted a literature review that supported the importance of explicitly teaching both executive function and visuo-spatial skills and suggested that these activities be integrated into elementary school curricula because of their ability to enhance young children’s learning of mathematics. Similarly, Honore and Noel (2017) conducted a study on a class of 34 kindergarten students to determine the impact of working memory training on counting, Arabic comparison, and calculation skills. Evidence gathered from the pre-test, training tasks, post-test, and maintenance test demonstrates that these skills (when taught in isolation) did not improve students’ numerical or arithmetic abilities. This research suggested that working memory training and mathematical skills must be taught together for math achievement to increase (Honore & Noel, 2017).

**Visual Processes**

Research has revealed that there is a need in mathematics instruction for the explicit introduction of visualization and concept mapping. Kaur (2018) developed a tool for teaching mathematics based on the concrete-pictorial-abstract approach. In this study, five teachers were selected for their leadership qualities and educational
experience. They used the Model method to teach a math unit to their students, and participated in an interview where they spoke to the effectiveness of this method. Based on the data collected from the students, Kaur concluded that the model method increased student math achievement. Similarly, Zhang’s study (2017) found that direct instruction with a concrete-pictorial-abstract sequence led to increased scores on geometry assessments. Together, these two researchers’ findings support the idea that moving students from concrete objects, to representations, then to abstractions is a logical instructional sequence that ultimately leads to increased learning.

The ability to visualize relationships among numbers in a word problem has been linked to student achievement in math (Kaur, 2018; Wilkerson-Jerde & Wilensky, 2011; Xin, 2018). Kaur’s Model Method (2018) enabled students to visually represent relationships between numbers in word problems, which is the key factor in the success of his program. Xin (2018) further supported Kaur’s earlier research by testing his Model Method on three third grade students with learning disabilities. These students all received below 40% on a word problem solving pre-test, and all scored above 90% after the intervention. After using the Model Method, these students experienced significant improvement. Finally, Wilkerson-Jerde and Wilensky (2011) described that a key component of mathematical learning is the ability to express relationships between mathematical ideas.

All this research supports the idea that math is composed of underlying processes and patterns, and real understanding of math skills comes from the ability to recognize these patterns. This is also consistent with the findings of Delisio, Bukaty, and
Taylor (2018) that schema-based instruction, which uses visual representations to identify problem structures, can have a positive impact on the problem-solving skills of students with disabilities. These researchers designed an intervention package featuring the KNWS graphic organizer and used the intervention on 84 students in inclusive fourth and fifth grade classrooms. Forty-seven of these students were SWD, and three students were identified as having ASD. The results of this study found that the intervention did help some students, but did not help all students on all problem types. They challenge educators to consider more individualized instruction in strategies for SWD (Delisio, Bukaty, & Taylor 2018).

**Socio-Emotional Processes.**

Several researchers support the claim that there must be an emotional connection to the material for real learning to occur (Arhin, 2015; Immordino-Yang & Damasio, 2007; McGregor, 2014). Black and Wiliam describe the process of formative assessment as being “deeply social and personal” (Black & Wiliam, 1998, p. 88). In a literature review of neuro-psychological studies, Immordino-Yang and Damasio (2007) state that emotion has a significant influence on children’s decisions to apply learning in other areas. Other researchers have focused on the effects of various instructional strategies, not only on math achievement but also on student’s attitudes towards math, supporting the long-held belief of teachers that attitude and achievement are inextricably linked (Arhin, 2015; McGregor, 2014). For example, Arhin conducted a study on the impact of performance driven instruction on the attitudes and achievement of 400 ninth grade students. The students were divided into two instructional groups, with
one group being given the performance assessments. There was a statistically significant
difference in the post-test scores of the two groups, supporting that performance
assessments lead to increased learning. In addition, the researcher concluded based on
qualitative observations that students were more engaged in the performance
assessment-driven learning and expressed more positive attitudes towards mathematics
than the students in the control group (Arhin, 2015).

**Instructional Strategies**

After laying a foundation of cognitive science principles, it is necessary to
examine how these principles can be translated into classroom practice. This section will
examine specific strategies for implementing the cognitive processes previously
described.

**Strategies for Foundational Processes**

The foundational processes refer to the movement of students from an initial
learning experience to understanding and application of a skill to transfer of the skill to
other contexts. The foundational processes provide the structure for an overall
framework for learning, and there are two instructional methods that follow this
movement pattern: schema-based instruction and inquiry-based learning.

**Schema-based instruction.** This instructional framework of schema-based
instruction is founded on the idea that problems can be divided into categories based on
their underlying problem structure (Kaur, 2018; Xin, 2018). Griffin, Gagnon, Jossi, Ulrich,
and Myers (2018) conducted a study that found that schema-based instruction was
highly effective in improving the problem-solving performance of students in fourth and
fifth grade classes at a rural school. After the intervention of schema-based instruction, these 42 students showed a statistically significant increase in scores in the area of word problem solving. In the words of these researchers, “The intent is to support students’ conceptual understanding and procedural fluency” (Griffin, et al., 2018, p. 151). In other words, schema-based instruction moves the student from an initial experience with a word problem to an understanding of that problem, then to an application of skill in solving the problem.

**Inquiry-based learning.** Another instructional framework that moves students from an initial experience to transfer of skill is inquiry-based learning. This method makes use of so-called “ill structured tasks” and open-ended questions to engage students in a process where they are driving the learning that happens in pursuit of a mathematical goal (McGregor, 2014). In his study on the effects of inquiry-based learning in a high-school classroom, McGregor found that this method had a positive effect on both the attitudes and achievement of students in mathematics. He conducted a qualitative study with 26 students between the ages of 15 and 17. The students were instructed in an inquiry-based, collaborative environment, and then interviewed for their thoughts on the instruction. The researcher concluded that an inquiry-based environment challenges students’ beliefs about math and improves their self-efficacy (McGregor 2014).

Similarly, Magee and Flessner (2012) describe the positive effects of using inquiry-based learning with pre-service teachers at the undergraduate level. These researchers conducted a qualitative study in which they selected 49 pre-service teachers
and engaged them in a series of inquiry-based experiences. The researchers kept reflective journals on the teaching experience, collected student artifacts, and sought student feedback to measure the success of their teaching. They found that after participated in inquiry-based experiences themselves, the pre-service teachers became more open to inquiry-based teaching despite previous misgivings about this method (McGregor, 2014). Inquiry-based learning is a student-centered approach to the foundational processes of learning.

**Strategies for Memory Processes**

The memory processes involve the research related to specific memory functions: executive functions, retrieval, and visuo-spatial skills. The visual aspect of these memory processes will be discussed in more detail in the following section. However, there are specific instructional strategies that can be used to train students in executive function and retrieval skills.

**Modeling of executive function.** Executive function is the process by which the learner focuses attention on a task at hand and uses internalized strategies of planning and recall to follow a problem-solving process. Executive function is an important skill in math problem-solving that must be explicitly modeled (Xin, 2018; Zhang, 2017). In a study on the effects of a conceptual-based approach on the problem-solving abilities of elementary students, Xin (2018) demonstrates a teacher’s use of modeling in guiding students through schema-based instruction. Xin’s study focused on the effects of this practice on 3 third-grade students who had specific learning disabilities and had failed a high-stakes test. The teacher used direct questioning strategies to guide students the
individual steps of the math skills. Finally, the students demonstrated significant improvement after the intervention.

Zhang’s (2017) earlier study describes how a step-by-step modeling of a strategy is necessary to ensure that students have solid understanding of a mathematical process. Using a small group of four college students who had scored under the 30th percentile in two different math assessments, Zhang led the students through an intervention strategy that focused on working memory training and leading students step by step through the process of a new skill. This study demonstrated that explicit training in working memory processes resulted in significant improvement in math skills for students with geometry disabilities. Finally, Kim and Cameron (2016) agree that research supports the claim that explicit teaching of executive function skills is foundational to math achievement.

Retrieval practice. Fuchs and Powell (2014) collaborated with other researchers to determine the effectiveness of retrieval strategies on calculation and word-problem instruction. Their goal was to determine which method was more effective for developing pre-algebraic knowledge. These researchers examined the results of calculation intervention versus word problem intervention on more than 1300 students in second grade classrooms. They found that the word-problem intervention produced better results than the calculation intervention, but that both calculation skills and word-problem solving skills increased when retrieval practices were consistently used. This research supports Agarwal’s (2019) findings that retrieval practice conditions led to increased improvement for both factual knowledge skills and higher-order thinking
skills, but that the higher-order retrieval practice resulted in the greatest overall improvements to learning.

**Strategies for Visual Processes**

Visuo-spatial skills and visual working memory are processes that could be described as either memory processes or visual processes. Many researchers have found that the ability to visualize relationships among mathematic processes is strongly correlated to high math achievement (Kaur, 2018, Wilkerson-Jerde & Wilensky, 2011; Xin, 2018). Four instructional strategies for helping students to visualize mathematic relationships are discovering pattern structures, engaging in concept mapping, developing visual planning, and using manipulatives.

**Pattern structures.** Chesney and McNeil (2014) conducted a study to test their theory that operational thinking can impede learning because it prevents students from being able to focus on the underlying structures of problems and the relationships among the numbers. They found that students who practiced operational strategies of solving math problems performed lower that students who practiced relational strategies. This idea of understanding the pattern behind a problem is supported by research in schema-based instruction (Delisio, et al., 2018; Kaur, 2018; Powell & Fuchs, 2018; Xin, 2018). Additionally, a qualitative study on middle school students’ understanding of mathematical patterns and relationships found that students who approached mathematics from a conceptual orientation demonstrated better understanding of the meaning of mathematical expressions in area and perimeter
problems (Bishop, 1997). Mathematical pattern structures are the conceptual basis for visual representations.

**Concept mapping.** One visual representation of mathematical patterns is concept mapping. In concept mapping, students use visual tools to explore the relationships between mathematic concepts. Bishop (1997) describes how students in her study were asked to model problems with manipulatives and then represented the relationships in the problem symbolically. The results of her study suggest that experience exploring relationships among problems help students develop deeper understanding. More recently, Bot and Eze (2016) developed a study where they examined the effects of concept mapping on students’ trigonometry achievement. They describe concept mapping as a tool which helps students to “visualize the relationships between various concepts and their meanings” (Bot & Eze, 2016, p. 57). Students who participated in concept mapping performed statistically higher than those who did not. Past and present research agree that concept mapping is an important strategy for math improvement.

**Visual planning.** Like concept mapping, visual planning is a way of representing mathematical ideas. However, instead of visualizing the relationships among quantities, visual planning helps students to track the mental problem-solving process. Powell and Fuchs (2018) call this an “attack strategy.” They give several examples of mnemonic devices for helping students remember the process of problem-solving. For example: DOTS- detect the problem type, organize the information in a diagram, transform the diagram into an equation, and solve for the unknown (Powell & Fuchs, 2018, p. 33).
They cite several of their research studies that concluded the effectiveness of using attack strategies to increase math performance. However, Delisio, et al. (2018) describe the results of their study in which they examined the effects of using the KNWS graphic organizer with students in an inclusive classroom. They discovered that there was no significant difference in results of students who used the graphic organizer (which is similar to the attack strategy) verses students who did not. This might suggest that visual planning tools are not as effective as concept mapping tools when it comes to improving math performance.

**Manipulatives.** Teachers have been using manipulatives in their math classroom for many years. Laski, Jor’dan, Daoust, and Murray (2015) conducted a recent study where they examined the use of manipulatives against cognitive science principles to determine how teachers can make the most of these materials. Their key findings were that manipulatives must be used consistently over a long period of time, that they should begin as concrete representations and gradually become abstract, and that the relationship between the manipulative and the math concept should be explicitly explained (Laski, et al., 2015, p. 2-5). This is consistent with the cognitive science principles of retrieval (Fuchs, et al., 2014), the concrete-pictorial-abstract approach (Kaur, 2018; Xin, 2018), and explicit modeling (Zhang, 2017).

**Strategies for Socio-Emotional Processes**

The socio-emotional processes are those that relate to the connections between learning and attitudes. Immordino-Yang and Damasio (2007) believe that neuroscientific evidence for emotional processes has strong connections to the field of education, and
several researchers have also demonstrated a link between attitude and achievement in mathematics (Arhin, 2015; McGregor, 2014). Some key strategies that teachers can use to engage the socio-emotional processes in mathematics instruction are through the use of performance assessments, cooperative learning, instructive feedback, and standards-based grading.

**Performance assessments.** In a study that examined the effects of performance assessment instruction on high school students, Arhin (2015) found that performance assessment significantly improved both attitude and achievement. He selected 400 students in 9th grade and designed an intervention package that was based on performance-based instruction. On average, students scored 16% higher on the post-test, after the intervention. Arhin claims that this is because the performance assessment provided the students with positive classroom experiences. Similarly, a group of researchers studying the effects of Design Thinking Framework found that students who had a personal connection to mathematical tasks were highly engaged in the learning process (Bush et al., 2018). These students were participating in an authentic math experience where they were collaborating to design a prosthetic for a student at a neighboring school. This emotional connection to the material ultimately resulted in increased learning.

One concern of teachers in adopting a performance assessment approach to instruction is how they are going to collect and record data to demonstrate student progress (Flexer, Cumbo, Borko, Mayfield, & Marion 1995). These researchers selected 14 volunteer teachers and asked them to implement a performance assessment
approach in their classrooms. Although the teachers did begin to use more hands-on and problem-solving activities after the assessment, they also reported some major concerns about the feasibility of implementing this strategy (Flexer et al., 1995). The concern is that performance assessment is subjective, and not a reliable or valid way to assess student learning. However, researchers Brown, O’Gorman, and Du (1996) found that when they applied the FACETS model to a math performance assessment, the data supported both the validity and reliability of the assessment.

Performance assessment not only transforms student learning; it can have a strong impact on teachers as well. More recently, Althauser and Harter (2016) conducted a quantitative study where they examined the effects on teachers and students who participated in a holistic performance assessment program that combined economics and math to create real-world mathematical experiences for students. As a result of their study, the students experienced significant growth in mathematics knowledge, and the teachers improved in the area of economics as well (Althauser & Harter, 2016). A 1995 study on the effect of performance assessment on teaching found that teachers who implemented these assessments began using more hands-on activities and providing students with more opportunities for problem solving and reported that students had shown growth after participating in performance assessment-driven instruction (Flexer et al., 1995).

Wiggins and McTighe write that the key goal of teaching is transfer, and that instruction is most effective when it is personalized: “making learners feel that they are an important part of something larger than themselves” (Wiggins & McTighe, 2006, p.
27). The process of tapping into students’ emotions and giving them a greater purpose for learning is one of the implications of emotional neuroscience research that is directly transferrable to education (Immordino-Yang & Damasio, 2007).

**Cooperative learning.** An explicit way to incorporate socio-emotional processes in the classroom is using cooperative learning. Bot and Eze (2016) found that when students were engaged in cooperative learning processes, math achievement improved for their students. They selected 210 students from three secondary schools to study the effects of cooperative learning strategies on their performance. On average, students showed between a 4-6 percentage point increase from the pre-test to the post-test (Bot & Eze, 2016).

Small group instruction is one expression of cooperative learning. Researchers Thomas and Feng (2014) developed a study to test whether homogenous or heterogenous ability groupings would produce better results for students. They conducted their study on 16 students in a self-contained classroom. There was no statistical difference from pre-test to post-test scores of homogenous groupings to those of heterogenous groupings. Their findings support the idea that neither ability grouping is better than the other, but they advise teachers to consider their students’ attitudes toward the groupings, supporting the idea that students’ attitudes and perceptions of themselves and their peers can have an effect on their learning (Thomas & Feng, 2014).

**Instructive feedback.** The practice of giving instructive feedback is related to the practice of frequent, purposeful formative assessment (Black & Wiliam, 1998; Leahy et
al., 2005). The purpose of feedback, according to these researchers, is to “move learners forward” (Leahy et al., 2005, p. 20). Instructional feedback is related to socio-emotional processes because it is a deeply personal encounter; the teacher interacts with the student about their individual work to help them see how the work relates to established standards and what they could improve upon in their learning (Black & Wiliam, 1998).

**Standards-based grading.** There are many reasons that educators assign grades to students, but two specific reasons are related to socio-emotional processes: providing feedback and motivating students to achieve greater learning (Lehman et al., 2018). Recent developments in education have led many school districts and individual schools to switch to a standards-based grading system. Lehman et al. (2018) conducted a study to test the results of a standards-based grading system versus a traditional grading system on student achievement. Their results were in favor of standards-based grading, as students in these systems scored higher on the Scholastic Math Inventory than students in a traditional grading system. While this system is designed to take subjectivity out of the grading process, it demonstrates a foundation of socio-emotional learning processes as it seeks to clearly communicate expectations, and separates academic and nonacademic components, which encourages greater accountability on the part of both the student and the teacher (Lehman et al., 2018).

**Conclusion**

Cognitive science principles can be directly correlated to many research-based instructional practices. These practices can be used to design instruction in a way that
supports greater student learning and achievement and encourages students to develop more positive attitudes towards mathematics instruction. Overall, these principles and practices reflect the idea that teaching, learning, and assessment are connected processes that, when integrated, lead to increased achievement in all three areas.
CHAPTER III: RESEARCH APPLICATION

Connection to Research

In order to apply the research from the literature review, two unit plans were created for use in an elementary mathematics classroom. The unit plans demonstrate how this research can be used to teach math skills in an elementary classroom. Unit 1 was created for a class of second and third grade students working on the skill of rounding to the nearest ten. Unit 2 was created for a class of fourth grade students learning to solve 2 and 3 digit division problems.

The purposes of these units vary in skill type and instructional level, but they follow the same pattern. The pattern of these units is based on the foundational processes of schema-based instruction and inquiry-based learning (Kaur 2018; Magee & Flessner 2012; McGregor 2014; Xin 2018). Both units also include memory processes of modeling executive function strategies and retrieval practice and visual processes of pattern structures, concept mapping, visual planning, and manipulatives. Finally, both units include a performance assessment guide that engages socio-emotional processes of cooperative learning, instructive feedback, and standards-based grading. These strategies are all in accordance with the research above, and together provide an example of how teachers can practically apply these strategies in an elementary math classroom.

The unit plans integrate foundational processes into the teaching process following schema-based instruction and inquiry-based learning structures to lead students from an initial learning experience to application of a skill (Washburn 2010;
Both plans use a form of schema-based instruction, in which they are building students’ conceptual understandings of a specific problem type (Griffin et al., 2018; Kaur, 2018; Xin, 2018). In the performance assessment component, both units use ill-structured tasks to build students’ understandings of the math skill (McGregor, 2014).

Both unit plans also include a plan for the modeling of executive function strategies. Teachers will model the problem solving process for students, thinking out loud during the step-by-step skill demonstration (Zhang, 2017). The teacher can then use this process to engage in direct questioning as they lead students through their own applications of the skill (Xin 2018). During days 2-3 of the unit plans, students will engage in higher-order retrieval practice that will help them to develop procedural fluency with the skill (Agarwal, 2019; Fuchs, et al., 2014; Honore & Noel, 2017; Jamil & Ghazali, 2018; Kim & Cameron, 2016).

These unit plans also rely on visual processes to teach mathematics skills. These visual processes rely on the underlying pattern structure of particular problems (Delisio et al., 2018; Kaur, 2018; Powell & Fuchs, 2018; Xin, 2018). Each math skill includes a visual tool that serves as a symbolic representation of the mathematical concept (Bishop, 1997). By using the process questions to lead students through the creation of the visual tool, teachers are also engaging in a form of visual planning (Powell & Fuchs, 2018). In the initial introduction of the skill, teachers can use manipulatives to demonstrate the math concept, leading students through the concrete-pictorial-
abstract approach to mathematics learning—as they connect known information with known information (Kaur, 2018; Laski et al., 2015; Xin, 2018; Zandieh et al., 2011).

Finally, these unit plans utilize specific socio-emotional processes to engage students in learning (Immordino-Yang & Damasio, 2007). The performance assessment component provides students with real-world opportunities to utilize the math skill, emphasizing the personal connection to the math task (Bush et al., 2018; Wiggins & McTighe, 2006). When using the performance assessment, teachers can allow students to engage in cooperative learning to boost engagement and performance (Arhin, 2015; Bot & Eze 2016). These performance assessments include a rubric in order to maintain validity and reliability in the assessment process and to provide a way for teachers to keep record of student progress (Brown et al., 1996; Flexer et al., 1995). Throughout the unit, teachers can use the Assessment Guide to provide meaningful instructive feedback and strengthen the teacher-student relationship (Althauser & Harter, 2016; Black & Wiliam, 1998; Leahy et al., 2005; Thomas & Feng, 2014). Involving the students in the assessment will also motivate them to achieve greater learning (Lehman et al., 2018).

Application Materials

Appendixes A and M are examples of math unit planning guides that utilize a specific process for teaching and learning. Appendix A is designed for a second-grade math unit in rounding. Appendix M is designed for a fourth-grade unit in division. These planning guides follow the same format of identifying the skill, giving an example of an initial encounter with the problem type, describing the process to solve the problem,
and depicting a visual representation of the problem solving process (Chesney & McNeil 2014; Washburn 2010; Wilkerson-Jerde & Willensky, 2011).

Both units follow the same teaching structure. Appendixes B and M provide a schedule for the introduction, application, and assessment of the new math skill. This plan includes components of inquiry-based learning in the initial encounter with the problem (Chesney & McNeil 2014; McGregor, 2014; Wilkerson-Jerde & Willensky, 2011). The plan also incorporates retrieval practice, where students continue to practice the skill by engaging in effortful recall (Agarwal, 2019; Brown et al.; Fuchs & Powell, 2014).

Appendixes C and O give a lesson plan for the first day of the unit- the initial encounter. The teacher’s modeling of the problem and demonstration of the visual representation activates memory processes (Xin, 2018; Zhang, 2017). The teacher will also present the underlying concept of the skill, which will enable students to connect the skill to other skills and concepts they encounter (Kaur, 2018; Griffin et al., 2018; Magee & Flessner, 2012; McGregor 2014; Xin, 2018;).

As students are given time for application, the teacher will provide immediate feedback on their process. Appendixes D, E, F, and P, Q, R, are examples of student application problems that can be used in their practice of the new skills. Appendixes G and S are feedback guides that the teacher can use to address student understanding of new skills, and also lists possible misconceptions students might have as they learn. These feedback guides also include a rubric that the teacher can use to assess student understanding of the skill. As students work to master the new skill, the teacher provides feedback about their performance based on the rubric. The rubric includes
student-friendly language and titles so that the students can engage with the teacher about where they are at in the learning process in a purposeful formative assessment (Black & Wiliam, 1998; Leahy, et al., 2005).

Finally, the unit plans include a performance assessment component as students integrate the new skill into real-world situations. Appendixes H and T serve as a guide for teachers to use in determining their level of understanding after three days of application practice. The student will then be given a performance task consistent with their level of mastery.

Appendixes I, J, K, and L are four performance assessment options for Unit 1. They present students with different opportunities to use the skill of rounding in a real-world setting, from determining how much money to take to the store to planning a menu for a cookout with friends. Appendixes U, V, W, and X are the performance assessments designed for Unit 2, all of which are different tasks that deal with dividing up pizzas for a class party. When the students have finished the performance tasks the teacher would refer to the assessment guide and rubric in Appendixes H and T to assess their overall level of mastery with the targeted skill.
CHAPTER IV: DISCUSSION AND CONCLUSION

Summary of Literature

Learning is only useful if it becomes permanent. (Washburn, 2010; Wiggins & McTighe, 2006;). Therefore, educators and researchers seek to discover strategies that will lead to permanence in learning. Effective teaching aligns with cognitive science principles. Learning, teaching, and assessment are interrelated processes that affect and inform each other (Arhin, 2015; Black & Wiliam, 1998; Flexer et al., 1995; Leahy et al., 2005; Washburn, 2010). This literature review identified four types of cognitive processes evident in effective learning strategies: foundational processes, memory processes, visual processes, and socio-emotional processes.

First, learning becomes permanent when it follows a pattern of foundational processes. These processes flow in order from an initial encounter with a new concept, to understanding, to application, and finally to an eventual transfer of skills (Washburn, 2010; Wilkerson-Jerde & Wilensky, 2011). This also reflects the idea of conceptual blending, wherein learners combine new information with known information (Zandieh et al., 2011). Schema-based instruction and inquiry-based learning are patterns for unit planning which follow this process and have proven to be effective strategies for teaching mathematics because of their focus on building authentic learning experiences and examining the underlying structure and relationships of math problems (Griffin, et al., 2018; Kaur, 2018; Magee & Flessner, 2012; McGregor, 2014; Xin, 2018).
Second, explicit instruction in memory processes leads to greater utility. Learning becomes permanent when teachers activate specific memory processes, such as executive function and retrieval practice. (Agarwal, 2019; Brown et al., 2014; Honore & Noel, 2017; Jamil & Ghazali, 2018; Kim & Cameron, 2016; Zhang 2017). In mathematics, a teacher’s explicit modeling of problem-solving strategies and direct questioning related to the individual steps of a math skill can lead students to greater understanding (Xin, 2018; Zhang, 2017). Retrieval practice has also been found to be an effective strategy for mathematics instruction, especially when the retrieval practice occurs in spaced-out sessions and involves higher-order thinking (Agarwal, 2019; Brown et al., 2014; Fuchs & Powell, 2014).

Third, the use of visual strategies for instruction enhances student understanding. Learning becomes permanent when students are engaged in visual processes (Delisio et al., 2018; Kaur 2018; Wilkerson-Jerde & Wilensky, 2011; Xin, 2018). Strategies for examining relationships among concepts (such as visualizing pattern structures, concept mapping) have proven to lead to greater learning. (Bishop, 1997; Chesney & McNeil, 2104). Manipulatives can also be an effective strategy for activating visual processes in mathematics learning (Laski et al., 2015). Specifically, manipulatives are useful when they begin as concrete representations that gradually become more abstract as comprehension deepens Kaur, 2018; Laski et al., 2015; Xin, 2018).

Finally, socio-emotional connections increase student engagement. Learning becomes permanent when students are emotionally invested in the material (Arhin, 2015; Immordino-Yang & Damasio, 2007; McGregor, 2014). Performance assessments
have been linked to both improved attitudes and achievements in mathematics because of their ability to provide students with authentic experiences (Althauser & Harter, 2016; Arhin, 2015; Bush, et al., 2018). These assessments have also been determined to be both valid and reliable methods of assessment (Brown et al., 1996). Relational strategies, such as cooperative learning and quality instructive feedback can have a significant impact on learning (Black & Wiliam, 1998; Bot & Eze 2016; Leahy et al., 2005; Thomas & Feng, 2014). A strategy for achieving quality instructive feedback is the use of standards-based grading, which has been demonstrated to encourage greater accountability for both the teacher and the student (Lehman, De Jong, & Baron, 2018).

Effective teaching utilizes the foundational processes, memory processes, visual processes, and socio-emotional processes. These processes provide a framework for mathematics instruction that will lead in an increase in student engagement and achievement. In the field of mathematics, educators seek to develop skills of mathematical thinking in students that will enable them to interpret and solve new problems using the skills that they have internalized. Ultimately, these cognitive processes work together to ensure that the concepts and skills learned in class become permanent and are transferable to other contexts.
Limitations of the Research

Research for this literature review was limited to the field of mathematics instruction. Topics and search terms included recent trends in the field of education, such as: “inquiry-based learning,” “performance assessment,” “standards-based grading,” “schema-based instruction,” “visual mathematics,” “patterns in mathematics,” and “concept mapping.” The search procedures also included a focus on cognitive science as it relates to the field of mathematics, using terms such as “conceptual blending,” “retrieval practice,” “executive function,” and “working memory skills.” Specifically, the search included literature that focused on these topics in the field of elementary education.

The research process was specifically focused on gathering peer-reviewed articles and studies that demonstrated specific learning strategies and their effects on a group of students. The research included both quantitative and qualitative studies. Some literature reviews were chosen based on their relevance to research and their summaries of qualitative and quantitative research studies in mathematics.

The question that drove this research was “What are the best cognitive science-based strategies for teaching mathematics at the elementary level?” Originally, the research was focused on mathematics instruction for elementary students. However, the pool of available research was limited in this area, so the search was expanded to include research at the junior high, high school, and adult learning levels. In the end, the research was compiled and applied in the creation of instructional materials for teachers.
of mathematics at the elementary level. Another limitation of the research was that the studies tended to represent a small sample size.

**Implications for Future Research**

The next phase of research in mathematics instruction should focus on the examination of specific instructional strategies and their effects on elementary students’ math achievement. Of 22 studies selected for this research project, only 8 focused on the area of elementary mathematics. Quantitative studies provide compelling evidence to adopt new instructional strategies, and it is necessary to expand this research to see how it affects instruction at the elementary level. It should be noted that quantitative research studies should include large sample sizes of elementary students, in order to collect more reliable data on the effect of instructional strategies. These studies should also examine the effects of specific instructional strategies on students with disabilities at the elementary level.

Additionally, future research should focus on the development of tools that can be used by elementary teachers to provide effective mathematics instruction to their students. This research suggests that the use of performance assessments and visual strategies can increase mathematics achievement. These performance assessments will need to be tested for validity and reliability. While the use of manipulatives is a prevalent classroom instructional practice, these manipulatives should be adapted to abstract visual representations that teachers can use to develop students’ understanding of problems as they move from concrete to abstract understanding.
Implications for Professional Application

The guiding principle that drove this research was that the pattern of teaching should follow the pattern of learning. By looking to the way children learn, teachers can design more effective instructional units. This research sheds light into the process that learners follow, specifically when understanding and applying new math skills. The application materials are an example of how this process can be adapted to fit a variety of lessons at different skill levels.

It is important to note that not all learners arrive at learning the same way. Conceptual blending, an important step in the path to understanding, is the process of combining new information with known information (Washburn 2010; Wilkerson-Jerde & Wilensky, 2011; Zandieh et al., 2011). But not all learners begin with the same amount of known information. Therefore, as teachers plan lessons and assessments for their students, they need to take multiple levels of learning into account.

Ultimately, the approach suggested in this paper is one of individualization. The emphasis on engaging in differentiated learning, meaningful instructive feedback, standards-based grading, and performance-based assessment is a motivated by the desire to see all students grow in their learning. These various instructional strategies are all tools that can be used to engage in student-centered instruction that meets the needs of a diverse group of learners.
Conclusion

Much work has been done to move students towards greater conceptual understanding of mathematics concepts. But there is still more to be done. Xin describes that there is still “a critical need for a paradigm shift in mathematics instruction toward mathematics model-based problem solving that emphasizes the overarching conceptualization of mathematical relations” (Xin 2018). It is my sincere hope that other educators can benefit from this research by applying these strategies and philosophies in their own classrooms. The science has spoken; now let the artists create.
References


*SAGE Open, 5*(2), 1-8. doi:10.1177/2158244015589588


McGregor, D., & Mathematics Education Research Group, of Australasia. (2014). Does inquiry based learning affect students' beliefs and attitudes towards mathematics?
Mathematics Education Research Group of Australasia, Retrieved from


http://www.ascd.org/portal/site/ascd/menuitem.459dee008f99653fb85516f762108a0c/

doi:10.1007/s11858-018-1002-9


Appendix A
Math Unit Planning Guide: Sample 1

Skill Overview

**Skill:** The student will round a number to the nearest 10.

**Underlying Concept:**
Numbers are evenly spaced.

**Related Skills:**
- Counting
- Estimation
- Measurement

**Problem Solving Process:**
1. What is the number?
2. What tens are on each side?
3. What is the midpoint between the tens?
4. Is the number above, below, or at the midpoint?
5. Which ten is the number closer to?

**Visual Representation:**

![Number Line](image)
Appendix B

Math Unit Planning Guide: Sample 1

Daily Plan

Day 1:
- Students work to solve initial encounter with problem.
- Teacher models problem-solving process.
- Students begin working on applications of problems.

Days 2-3:
- Students solve applications of problems.
- Teacher gives immediate instructive feedback on process.

Day 4-5:
- Teacher introduces performance assessment.
- Students begin working on performance assessment.
- Teacher provides instructive feedback on process.

Day 6:
- Students present performance assessments.
- Teacher conferences with students to discuss mastery of skill.

Retrieval Practice:
- After the unit's completion, the teacher will continue to engage the students in effortful recall by providing opportunities for the student to practice the skill.
Appendix C

Math Unit Planning Guide: Sample 1

Day 1: Initial Encounter

**Encounter:**

Ask the students the following questions and give them time to find solutions.
- Is 32 closer to 30 or 40?
- Is 67 closer to 60 or 70?
- Is 45 closer to 40 or 50?

**Discussion:**

Ask the students the following questions:
- What are your answers?
- What process did you use to solve these?
- Which one was the hardest?
- Do you disagree with each other’s answers? Why or why not?

**Connection:**

Introduce the underlying concept to the students: Numbers are evenly spaced. Ask them to consider how the activity showed them that this is true, or how they can tell that this is true. Have the students draw a diagram to show that numbers are evenly spaced.

**Direct Instruction:**

Introduce the steps of the problem solving process: identify the number, identify the tens, identify the midpoint, locate the number in reference to the midpoint, identify the closer ten. Use the process to model solving each of the initial encounter problems. As you model the process, think out loud to show the students how the questions can help them solve the problem.

**Guided Practice:**

Give the students Application Problems Day 1.
Math Unit Planning Guide: Sample 1

Application Problems Day 1

1. Use the number line to round the number 37.

2. Use the number line to round the number 42.

3. Use the number line to round the number 75.

4. 67 is between what two tens numbers?

5. 52 is between what two tens numbers?

6. 28 is between what two tens numbers?

7. Use the number line to round the number 81.

8. Use the number line to round the number 17.
Appendix E

Math Unit Planning Guide: Sample 1

Application Problems Day 2

1. Use the number line to round the number 46.

   ![Number Line for 46]

2. Use the number line to round the number 29.

   ![Number Line for 29]

3. Use the number line to round the number 52.

   ![Number Line for 52]

4. Round the number 53.

5. Round the number 71.

6. Round the number 35.
Appendix F

Math Unit Planning Guide: Sample 1

Application Problems Day 3

1. Is 7 closer to 0 or 10?

2. Is 12 closer to 10 or 20?

3. Is 85 closer to 80 or 90?

4. Round the number 43.

5. Round the number 98.

6. Round the number 5.

7. Round the number 27.

8. Round the number 16.
Appendix G

Math Unit Planning Guide: Sample 1

Feedback Guide

Formative Assessment:

As students work to apply the process for the skill, give them feedback about where they are in the phases of mastery using the rubric below

<table>
<thead>
<tr>
<th>GUIDE</th>
<th>EXPLORER</th>
<th>TRAILBLAZER</th>
<th>SEEKER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student has exceeded mastery for the skill by coming up with new strategies, integrating with other skills, etc.</td>
<td>Student is able to perform the skill independently with accuracy and efficiency.</td>
<td>Student is able to perform the skill with help or if looking at an example.</td>
<td>Student is not able to perform the skill.</td>
</tr>
</tbody>
</table>

Possible Misconceptions:

1. Students may need help understanding that 0 and 100 are also "tens numbers" in this context.

2. Sometimes students who are new to this skill place a number between the wrong "tens," for example, saying that 25 is between 10 and 30 rather than 20 and 30.

3. Students may need to be reminded that although the number 5 is a midpoint between two tens, it is rounded up to the larger ten number.
Math Unit Planning Guide: Sample 1

Performance Assessment Guide

Determining Placement:
When assigning the performance assessment, work with the students to select an option for assessment that is consistent with their level of mastery by Day 3 of application.

<table>
<thead>
<tr>
<th>GUIDE</th>
<th>EXPLORER</th>
<th>TRAILBLAZER</th>
<th>SEEKER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student has exceeded mastery for the skill by coming up with new strategies, integrating with other skills, etc.</td>
<td>Student is able to perform the skill independently with accuracy and efficiency.</td>
<td>Student is able to perform the skill with help or if looking at an example.</td>
<td>Student is not able to perform the skill.</td>
</tr>
</tbody>
</table>

Process:
Give the students the materials to complete their performance assessment. Allow them to work on the assessment for 2 days. As they work, continue to engage in instructive feedback. On the final day of the unit, allow each student to present their performance assessment. Assess their work using the above rubric.
Appendix I

Differentiated Performance Assessment: Sample 1

Option 1: Guide

Task:

You are planning a cookout for some friends. You are planning to have 25 people come, and your budget is $100. Design a menu plan for your cookout. Estimate the total cost of your groceries. Here are the prices for some cookout items at the store:

- Hamburgers: 4 for $5.00
- Hotdogs: 8 for $4.00
- Hamburger Buns: 8 for $2.00
- Hotdog Buns: 10 for $2.00
- Ketchup: $1.75
- Mustard: $1.25
- Relish: $1.75
- Mayonnaise: $2.25
- Lemonade: 25 servings for $2.50
- Coke: 10 cans for $5.00
- Marshmallows: 30 for $2.25
- Graham Crackers: 15 for $3.00
- Mini Chocolates: 30 for $5.00
- Watermelon: 2 for $5.00
Appendix J

Differentiated Performance Assessment: Sample 1

Option 2: Explorer

Task:
Your neighbor has asked you to run to the store to get her some groceries, but she is not sure how much cash to give you. Here is her list, and here is this week’s ad from the newspaper. About how much money do you need?
Differentiated Performance Assessment: Sample 1

Option 3: Trailblazer

Task:
Your neighbor has asked you to run to the store to get her some groceries, but she is not sure how much cash to give you. Here is her list, and here is this week’s ad from the newspaper. About how much money do you need?
Appendix L

Differentiated Performance Assessment: Sample 1

Option 4: Seeker

Task:

Your neighbor has asked you to run to the store to get her some groceries. Here is her list. About how much money do you need to take?

1 Box Cereal $2.00
1 Gallon Milk $4.00
1 Package Cookies $4.00
1 Bag Salad Mix $1.00
1 Bunch of Bananas $2.00
1 Watermelon $2.00
1 Package Bacon $6.00
1 Can of Coffee Beans $2.00
1 Carton of Eggs $1.00
1 Bag of Chips $2.00
Math Unit Planning Guide: Sample 2

Skill Overview

Skill: The student will solve 2 and 3 digit division problems.

Underlying Concept:
A whole number can be broken up into smaller equal groups.

Related Skills:
Addition & Subtraction
Counting Strategies
Multiplication

Problem Solving Process:
1. How many groups are there?
2. How many items can be taken from the whole number?
   (repeat until the whole can't be broken down more)
3. How many items are in each group?
4. How many items are left in the whole?

Visual Representation: 473 ÷ 4 = 118 r. 1

<table>
<thead>
<tr>
<th>100</th>
<th>100</th>
<th>100</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

- 400
- 40
- 33
+ 8
- 32

118 r. 1
Math Unit Planning Guide: Sample 2

Daily Plan

Day 1:
Students work to solve initial encounter with problem.
Teacher models problem-solving process.
Students begin working on applications of problems.

Days 2-3:
Students solve applications of problems.
Teacher gives immediate instructive feedback on process.

Day 4-5:
Teacher introduces performance assessment.
Students begin working on performance assessment.
Teacher provides instructive feedback on process.

Day 6:
Students present performance assessments.
Teacher conferences with students to discuss mastery of skill.

Retrieval Practice:
After the unit's completion, the teacher will continue to
engage the students in effortful recall by providing
opportunities for the student to practice the skill.
Math Unit Planning Guide: Sample 2

Day 1: Initial Encounter

**Encounter:**

Ask the students the following questions and give them time to find solutions.

- How can 9 candies be given to 3 people?
- How can 21 candies be given to 3 people?
- How can 80 candies be given to 3 people?

**Discussion:**

Ask the students the following questions:

- What are your answers?
- What process did you use to solve these?
- Which one was the hardest?
- Do you disagree with each other's answers? Why or why not?

**Connection:**

Introduce the underlying concept: A whole number can be broken up into smaller equal groups. Ask the students if they discovered this in the initial encounter. Have the students draw a series of pictures to demonstrate how a whole number can be broken up into smaller groups. Alternatively, allow the students to use manipulatives to demonstrate this concept.

**Direct Instruction:**

Introduce the steps of the problem solving process: identify the number of groups, break down the number, add the numbers in each group, count the leftovers. Use the process to model solving each of the initial encounter problems. As you follow the process, explain your thinking to the students, being sure to model each step of the process.

**Guided Practice:**

Give the students Application Problems Day 1.
Math Unit Planning Guide: Sample 2

Application Problems Day 1

1. Use the tool to solve $652 \div 5$

```
[Diagram showing division of 652 by 5]
```

2. Use the tool to solve $345 \div 3$

```
[Diagram showing division of 345 by 3]
```

3. Solve $739 \div 4$
Math Unit Planning Guide: Sample 2

Application Problems Day 2

How many groups would you create for these problems?

1. $798 \div 5$ ______

2. $234 \div 3$ ______

3. $476 \div 8$ ______

Use a separate sheet of paper to solve the following problems.

1. Solve $347 \div 3$

2. Solve $250 \div 2$

3. Solve $653 \div 5$

4. Solve $508 \div 4$

5. Solve $885 \div 7$
Math Unit Planning Guide: Sample 2

Application Problems Day 3

Use a separate sheet of paper to solve the following problems.

1. Solve $439 \div 5$

2. Solve $360 \div 3$

3. Solve $845 \div 6$

4. Solve $315 \div 4$

5. Solve $622 \div 2$
Appendix S

Math Unit Planning Guide: Sample 2

Feedback Guide

Formative Assessment:
As students work to apply the process for the skill, give them feedback about where they are in the phases of mastery using the rubric below

<table>
<thead>
<tr>
<th>GUIDE</th>
<th>EXPLORER</th>
<th>TRAILBLAZER</th>
<th>SEEKER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student has exceeded mastery for the skill by coming up with new strategies, integrating with other skills, etc.</td>
<td>Student is able to perform the skill independently with accuracy and efficiency.</td>
<td>Student is able to perform the skill with help or if looking at an example.</td>
<td>Student is not able to perform the skill.</td>
</tr>
</tbody>
</table>

Possible Misconceptions:
1. Students may need help understanding what to do with the “leftover” number. A visual demonstration might be helpful in this instance.
2. Students might need reminders in counting by 25s or 20s, which will be helpful for dividing up hundred’s numbers.
3. Continue to work with students on counting strategies and multiplication facts to support their understanding of this skill.
Math Unit Planning Guide: Sample 2

Performance Assessment Guide

Determining Placement:

When assigning the performance assessment, work with the students to select an option for assessment that is consistent with their level of mastery by Day 3 of application.

<table>
<thead>
<tr>
<th>GUIDE</th>
<th>EXPLORER</th>
<th>TRAILBLAZER</th>
<th>SEEKER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student has exceeded mastery for the skill by coming up with new strategies, integrating with other skills, etc.</td>
<td>Student is able to perform the skill independently with accuracy and efficiency.</td>
<td>Student is able to perform the skill with help or if looking at an example.</td>
<td>Student is not able to perform the skill.</td>
</tr>
</tbody>
</table>

Process:

Give the students the materials to complete their performance assessment. Allow them to work on the assessment for 2 days. As they work, continue to engage in instructive feedback. On the final day of the unit, allow each student to present their performance assessment. Assess their work using the above rubric.
Appendix U

Differentiated Performance Assessment: Sample 2

Option 1: Guide

Task:

Your class is having a party. There are 12 students, 2 parent volunteers, and 1 teacher. Your teacher says she will order 6 pizzas. The pizzas can be cut into either 10, 8, or 6 slices each. What is the best way to cut the pizzas so that there will be the fewest amount of slices leftover?
Appendix V

Differentiated Performance Assessment: Sample 2

Option 2: Explorer

Task:

Your class is having a party. There are 12 students, 2 parent volunteers, and 1 teacher. Your teacher says she will order 6 pizzas. The pizzas can be cut into either 8 or 6 slices each. What is the best way to cut the pizzas so that there will be the fewest amount of slices leftover?
Differentiated Performance Assessment: Sample 2

Option 3: Trailblazer

Task:

Your class is having a party. There will be 20 students, 2 parent volunteers, and 1 teacher. How many slices of pizza should each person get? How many will be left over?
Appendix X

Differentiated Performance Assessment: Sample 2

Option 4: Seeker

Task:

Your class is having a party. There will be 15 students, 2 parent volunteers, and 1 teacher. How many slices of pizza should each person get?